THE DYNAMICS OF ROSSBY AND INERTIAL WAVES IN THE IONOSPHERE WITH AN INHOMOGENEOUS ZONAL WIND: AMPLIFICATION AND MUTUAL TRANSFORMATIONS

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(Received: 12.01.05; accepted: 17.05.05)

Abstract

The generation and further dynamics of planetary magnetized Rossby waves and inertia waves are investigated in the rotating dissipative ionosphere in the presence of a smooth inhomogeneous zonal wind (shear flow). Magnetized Rossby waves appear as a result of the interaction of the medium with the spatially inhomogeneous geomagnetic field and are ionospheric manifestation of usual tropospheric Rossby waves. An effective linear mechanism responsible for the intensification and mutual transformation of Rossby and inertia waves is found. For shear flows, the operators of the linear problem are nonselfconjugate and therefore the eigenfunctions of the problem are nonorthogonal and can hardly be studied by the canonical modal approach. Hence it becomes necessary to use the so-called nonmodal mathematical analysis which has been actively developed in recent years. The nonmodal approach shows that the transformation of wave disturbances in shear flows is due to the nonorthogonality of eigenfunctions of the problem in the conditions of linear dynamics. Thus there arise a new degree of freedom and a new way for the evolution of disturbances in the medium. Using the numerical modeling, we illustrate the peculiar features of the interaction of waves with the background flow, as well as the mutual transformation of wave disturbances in the **D**, **E** and **F**-regions of the ionosphere. It is established that the presence of a geomagnetic field, Hall and Pedersen currents in the ionospheric medium improve the interaction and mutual energy exchange between waves and a shear flow.

Key words and phrases: Shear flow, inhomogeneous geomagnetic field, Rossby wave transformation.

AMS subject classification: 3.35.G; 52.35.Bj; 52.35. Dm; 52.35.M

1. Introduction

Large-scale wave motions play an important role in the processes connected with energy balance and atmosphere and ocean circulation. As a simple theoretically possible kind of large-scale motion in the ionosphere, we can name geostrophic motion, i.e. motion in whose dynamic equations the horizontal pressure gradient and the Coriolis force have the same order, while the other terms are negligibly small. As the classical investigations in this area showed [Rossby, 1938; Obukhova, 1949; Kibel, 1955], real large-scale motions are characterized by a continuous process of adaptation to geostrophic motion. For synoptic practice, the geostrophic approximation gives a satisfactory accuracy especially for lower atmospheric layers (except the Earth's boundary layer, frontal surfaces and jet flows) [Gandin et al., 1952; Holton, 1976].

However, as different from lower atmospheric layers, when studying the dynamics of large-scale planetary processes in the ionosphere, it is necessary to take into account the inhomogeneous and non-stationary properties of a wind process, a turbulent state of the lower ionosphere, and the influence of inhomogeneous electromagnetic forces. These factors, which are especially strongly pronounced because of a low density of the medium in the ionosphere and a relatively high conductivity of the ionospheric gas, may cause essential deflections of the real wind (usual Rossby planetary wave) from geostrophic motion. Hence the general ionospheric circulation has certain peculiarities that are not observed in the conditions of the troposphere.

A stationary problem on the existence of large-scale (planetary) Rossby waves (horizontal winds) in the ionosphere was for the first time discussed in [Dokuchayev, 1959] for the case of a rectilinear homogeneous flow of the medium in the geomagnetic field. It was found, that for theoretical investigation and interpretation of the dynamics of winds above 100 km, it is necessary to take into consideration possible deflections from the geostrophic wind, which arise under the action of electromagnetic forces. In the subsequent years, there appeared a number of other works [Bramley, 1967; Geisler, 1967; Khantadze, 1968; Khantadze, Sharikadze, 1969; Aburjania, Kahntadze, 2002 and others], in which consideration was given to nonstationary evolutions of wind structures in the conducting ionospheric medium under the influence of a spatially inhomogeneous geomagnetic field.

The action of a geomagnetic field leads, on the one hand, to the inductive damping of Rossby type planetary waves, which is connected with Pedersen or transverse (relative to a magnetic field) conductivity, and, on the other hand, to the gyroscopic effect caused by the Hall conductivity of the ionosphere and having an impact on disturbances like the Coriolis force. As a result of the joint action of the spatially inhomogeneous Coriolis force and the electrodynamic (connected with the geomagnetic field) force, in the ionosphere there may exist a new type of waves which physically differ from the usual Rossby wave and which are called magnetized Rossby or Rossby type waves.

In the above references and other earlier works, the Rossby wave or magnetized Rossby wave dynamics was studied, at best, in the presence of the constant homogeneous zonal wind. Hence the corresponding dynamic equations were solved by the canonical modal approach, i.e. by the spectral (Fourier or Laplace) expansion of wave values with respect to time.

However, many-year observations [Khantadze, 1973; Gossard, Hooke, 1975; Pedlosky, 1978; Kazimirovski, Kokourov, 1979; Kamide, 1980] show that the atmospheric and ionospheric layers always have spatially inhomogeneous zonal winds (shear flows) produced by a nonuniform heating of the atmospheric layers by solar radiation. In this connection, it becomes important to investigate the problem on generation and evolution of usual and magnetized Rossby waves at their interaction with the inhomogeneous zonal wind (shear flow).

The interest in shear flows exist, generally speaking, due to their occurrence both in the near-earth space (as has been mentioned above) and astrophysical objects (galaxies, stars, jet outbursts, the world ocean and so on) and in the laboratory and engineering equipment (oil and gas pipelines, plasma magnetic traps, magnetodynamic generators and so on). A flow velocity shear is a powerful source of various energy-consuming processes in a solid medium. Though these processes have been studied in the course of many years, their theoretical interpretation is difficult even in terms of linear approximation. The canonical (modal) investigation of linear wave processes (spectral expansion disturbances with respect to time followed by analysis of the eigenvalues) in shear flows does not take into account a highly important physical process, namely, the mutual transformation of wave modes [Reddy et al., 1993; Trefenthen et al., 1993].

A strict mathematical description of the peculiarities of shear flows revealed [Reddy et al., 1993] that in the case of canonical (modal) analysis of linear processes the operators figuring in dynamic equations are not self-conjugate [Trefenthen et al., 1993] and, as a result, the eigenfunctions of the problem are not orthogonal to each other – they strongly interfere with each other. One of the results of this fact consists in the following: even if all eigenfunctions decrease monotonically (exponentially) with respect to time (i.e. if the complex parts of all eigenfrequencies are negative), a particular solution might show a large relative growth on the finite time interval. Therefore analysis of individual eigenfunctions and eigenvalues does not help us to form a judgement about the linear stage of the evolution of shear flow disturbances. Thus, for a correct description of phenomena it becomes necessary to carry out accurate calculations of effects of the interference of eigenfunctions, which sometimes turns out to be the problem of insurmountable difficulty.

There also exists another so-called nonmodal analysis of linear processes in shear flows, which takes its origin in the time of Kelvin [1887]. With this approach, the modified initial problem (Cauchy problem) is solved by tracing the evolution of spatial Fourier-harmonics (SFH) of wave disturbances in time and not using any spectral expansion with respect to time [Graik et al., 1986; Farrell, Ioannou,1993; Chagelishvili et al., 1994; Chagelishvili et al., 1996; Kalashnik et al., 2004]. Being an optimal "language", the nonmodal approach much simplifies a mathematical description of the dynamics of shear flow disturbances and makes it possible to reveal the key phenomena (caused by the nonorthogonality of linear dynamics) which have escaped the notice in the case of modal analysis. A lot of unexpected new results have already been obtained within the framework of this approach. They include in particular the evolution of acoustic disturbances, an energy exchange between the corresponding SFH and a horizontal shear flow [Chagelishvili et al., 1994; Chagelishvili et al., 1996]; a new mechanism of linear transformation of waves in shear flows has been discovered [Chagelishvili et al., 1995; Chagelishvili et al., 1997].

Usually, when investigating the dynamics of Rossby type waves in the dispersed medium (atmosphere, ionosphere, ocean), in the corresponding closed system of hydro- or magnetohydrodynamic equations, we perform expansion with respect to the small parameter (Rossby parameter). This is in fact the averaging over a high-frequency inertial branch of oscillations and, as a result, we obtain the vortex transfer equation or the Charney-Obukhov equation [Charney, 1947; Obukhov, 1949] analyzed in most of the works dealing with the dynamics of Rossby type waves [Rossby, 1949; Gossard, Hooke, 1978; Monin, 1978; Pedlosky, 1978; Gill, 1982; Petviashvili, Pokhotelov, 1992; Nezlin, Snezhkin, 1993]. Such an approximation certainly excludes a possible occurrence of fast processes in the system and ignores a possible transformation of Rossby type waves to high-frequency gyroscopic waves in the presence of zonal shear flows (wind) and thereby may strongly distort the picture of wave processes in the atmosphere. Therefore such an approach closes the channel through which a greater part of energy of Rossby type waves is transferred.

We will show below that even in the case of a simple shear flow (smooth-inhomogeneous wind) the use of the Charney-Obukhov equation as a base model leads to ignoring the important process of energy exchange between high-frequency (inertial) waves and low-frequency (Rossby) ones. Here we actually mean the transformation of waves of a low-frequency branch to waves of a high-frequency branch, i.e. we can speak of an essential change occurring in the time scale of the wave process. The matter is that in shear flows, waves of various scales become connected: in equations describing their evolution and written with appropriate notation, there appear connected (linked) terms which for certain values of system parameters lead to intensive mutual transformations of modes.

In this paper we investigate the linear evolution of Rossby type waves in shear zonal flows (winds) in **D**, **E** and **F**-regions of the ionosphere. In dynamic equations, the disturbed magnetohydrodynamic values are represented through SFH. This corresponds to nonmodal analysis in the coordinate system which moves with the background wind. This spatial Fourier expansion allows us to replace, in the basic equations, the spatial inhomogeneity connected with the inhomogeneity of the basic zonal flow by the time-dependent inhomogeneity and to trace how the SFH of disturbances evolved in time.

2. Initial equations and the basic principles of nonmodal analysis

In this paper we are interested mainly in large-scale (planetary) wave motions in the ionospheric medium (consisting of electrons, ions and neutral particles), which have a horizontal linear scale L_h of order 10^3 km and higher, a vertical scale L_v of altitude scale order H_0 ($L_v \approx B_0$) and a time scale τ of half-day order and higher. It is such motions that are connected with global distributions of the ionospheric structure and its large-scale daily, seasonal, 27-day and other variations. According to experimental data [Ratclife, Weeks, 1960; Gossard, Hooke, 1975; Holton, 1975; Kazimirovski, Kokourov, 1979; Kamide, 1980], in ionospheric large-scale motions the relation of the characteristic vertical velocity V_v to the horizontal one V_h is small: $V_v/V_h \leq L_v/L_h < 10^{-2}$. The latter relation implies that large-scale motions in the ionosphere are mostly quasi-horizontal. The dynamic properties of such a medium are defined by the neutral component because of the fulfillment of the condition $N_{e,i}/N_n << l$ (where N_e, N_i, N_n are the concentration of electrons, ions and the neutral component, respectively). The presence of charged particles makes the considered medium electroconductive.

Among theoretically possible ionospheric large-scale wave motions we can single out a class of disturbances for which the effective Reynolds magnetic number is $R_{eff} \approx 4\pi\sigma_{eff} V \cdot L \cdot c^{-2} << 1$ (where σ_{eff} is an effective value of ionospheric conductivity, c is the light velocity, V and L are the characteristic velocity and disturbance sizes), which is sufficiently well fulfilled nearly up to the ionospheric F-layer [Khantadze, 1973; Kazimirovski, Kokourov, 1979; Kamide, 1980]. Hence for the lower ionosphere we can neglect the induced magnetic field $b \approx R_{eff}B$ and the vortical electric field $E_v\tilde{}R_{eff}(VB)$ that arises at the change of b. Thus for the considered class of wave disturbances the magnetic field can be assumed to be given and equal to the external spatially inhomogeneous geomagnetic field $\mathbf{B}_0(\mathbf{B} = \mathbf{b} + \mathbf{B}_0 \approx \mathbf{B}_0,\ \mathbf{E}_v \to 0)$. It satisfies the equations $div\mathbf{B}_0 = 0$, $rot\mathbf{B}_0 = 0$. Having such an induction-free approximation, it is sufficient to consider only currents j arising in the medium, while the magnetic field generated by these currents can be neglected. In that case, the action of the geomagnetic field $\mathbf{B}_0$ on the induction current $\mathbf{J}$ in the ionospheric plasma makes it necessary to take into consideration the ponderomotive force $\mathbf{j} \times \mathbf{B}_0$ in the well known equations of ionospheric dynamics (in addition to the pressure, Coriolis and viscous friction forces). The presence of this force not only modifies the geostrophic wind (because of Hall currents), but makes the deflect wind from the geostrophic wind due to the appearance of inductive deceleration (caused by Pedersen currents) in the Earth's ionosphere which is more essential than viscous decceleration [Dokuchayev, 1959; Gershman et al., 1984], especially in the F-region [Khantadze, 1973; Kamide, 1980].

It might seem that large-scale Rossby type disturbances in the ionosphere could be described by shallow water equations. However, when using these equations for atmospheric long-wave processes, the atmosphere is usually assumed to be barotropic and in reality, as is seen from synoptic maps, this assumption does not hold always. In [Petviashvili, Pokhotelov, 1992] it is shown that the system of shallow water equations should take into account the medium compressibility.

In the light of the above reasoning, the basic properties of a Rossby type planetary wave in the ionosphere should be considered by using as initial data the equation for the horizontal velocity $\mathbf{V}_\perp(V_x, V_y)$, where acceleration is defined by the pressure gradient, Coriolis force, volumetric electrodynamic force and viscous friction [Dokuchayev, 1959; Gossard, Hooke, 1975; Pedlosky, 1978; Kamide, 1980]:

$$\frac{\partial \mathbf{V}_\perp}{\partial t} + (\mathbf{V}_\perp \triangledown)\mathbf{V}_\perp = -\frac{\triangledown P}{\rho} - 2\left[\Omega_0 \times \mathbf{V}_\perp\right] + \frac{1}{\rho c}\left[\mathbf{j} \times B_0\right] + \nu \triangle_\perp \mathbf{V}_\perp, \quad (2.1)$$

the continuity equation [Petviashvili, Pokhotelov, 1992]

$$\frac{\partial \rho}{\partial t} + (\mathbf{V}_\perp \triangledown)\rho + \rho\gamma^{-1} div\mathbf{V}_\perp = 0 \quad (2.2)$$

and the medium state equation

$$\frac{\partial P}{\partial t} + (\mathbf{V}_\perp \triangledown)P + P div\mathbf{V}_\perp = 0. \quad (2.3)$$

Here P and $\rho = N_n M$ are the pressure and the density of the medium, M is the mass of an ion or a neutral particle (molecule), $\mathbf{g}$ is the gravity force acceleration, γ is the ratio of specific heats, ν is the kinematic viscosity, $\triangle_{\perp} = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is the horizontal Laplacian. The ponderomotive force $\mathbf{j} \times \mathbf{B}_0$ defines to a considerable extent the peculiar behavior of ionospheric motions [Aburjania, Khantadze, 2002; Aburdjania et al., 2004]. The inductive current j density is defined from the generalized Ohm's law for the ionosphere [Khantadze, 1973; Kamide, 1980; Gershman et al., 1984]:

$$\mathbf{j} = \sigma_{\|}\mathbf{E}_{d_{\|}} + \sigma_{\perp}\mathbf{E}_{d_{\perp}} + \frac{\sigma_H}{B_0}\left[\mathbf{B}_0 \times \mathbf{E}_d\right], \tag{2.4}$$

where the parallel conductivity (in the direction of the magnetic field $\mathbf{B}_0$) , the Pedersen or transverse conductivity (across $\mathbf{B}_0$) and the Hall conductivity are defined by the following expressions:

$$\sigma_{\|} = e^2 N\left(\frac{1}{m\nu_e} + \frac{1}{M\nu_{in}}\right),$$

$$\sigma_{\perp} = e^2 N\left(\frac{\nu_e}{m\left(\nu_e^2 + \omega_{Be}^2\right)} + \frac{\nu_{in}}{M\left(\nu_{in}^2 + \omega_{Bi}^2\right)}\right), \tag{2.5}$$

$$\sigma_{H} = e^2 N\left(\frac{\omega_{Be}}{m\left(\nu_e^2 + \omega_{Be}^2\right)} - \frac{\omega_{Bi}}{M\left(\nu_{in}^2 + \omega_{Bi}^2\right)}\right),$$

where e, m, $\nu_e = \nu_{ei} + \nu_{en}$, $\omega_{Be} = eB_0/m$ are the charge, mass, frequency of collisions of electrons with ions and neutral molecules and the cyclotronic frequency of electrons, respectively; ν_{in} and $\omega_{Bi} = eB_0/M$ are the corresponding ion values. Assuming that the ionosphere is quasi-neutral to a high accuracy degree, we have neglected the electrostatic part $E_e = -\nabla\Phi$ (Φ is an electrostatic potential) and the vortical part $\mathbf{E}_v$ of the electric field. Now, if we take into account the motion of the medium, then the electric field intensity in equation (2.4) is defined only by the dynamo-field [Dokuchayev, 1959; Khantadze, 1973; Kamide, 1980]

$$\mathbf{E}_d = \left[\mathbf{V} \times \mathbf{B}_0\right] \tag{2.6}$$

Since the length of planetary waves is comparable with the Earth's radius R, we investigate such notions in approximation of the β-plane, which was specially developed for analysis of large-scale processes [Gossard, Hooke, 1975; Pedlosky, 1978], in the "standard" coordinate system [Gandin et al., 1955; Holton, 1975]. In this system, the x-axis is directed along the parallel to the east, the y-axis along the meridian to the north and the z-axis vertically upwards (the local Cartesian system). The differentials dx,

dy, dz are related to the parameters of the spherical system of coordinates λ, θ, r by the following approximate formulas: $dx = R\sin\theta d\lambda$, $dy = -Rd\theta$, $dz = dr$. The velocties are respectively equal to $V_x = V_\lambda$, $V_y = -V_\theta$, $V_z = V_r$. Here $\theta = \pi/2 - \varphi$ is the colatitude, φ is the geographical latitude, λ is the longitude, r is counted from the center along the Earth's radius. In the sequel, we assume that $V_z = 0$ (by virtue of the above reasoning) and the geomagnetic field $\mathbf{B}_0$ (B_{0x}, B_{0y}, B_{0z}) is dipolar and has in the chosen coordinate system the following components [Dokuchayev, 1959; Khantadze, 1973]

$$B_{0x} = 0,\ B_{0y} = -B_e \sin\theta^{'}, B_{0z} = -2B_e\cos\theta^{'}, \tag{2.7}$$

where $B_e \approx 3,5\times 10^{-5}$Tesla (T) is the value of geomagnetic field induction at the equator. In this case, the total induction of the geomagnetic field is $B_0 = B_e\left(1 + 3\cos^2\theta^{'}\right)^{1/2}$ and $\theta^{'} = \pi/2 - \varphi^{'}$, where $\varphi^{'}$ is the geomagnetic latitude. In the same coordinate system, the components of the angular velocity vector of the Earth's rotation $\Omega_0\,(\Omega_{0x}, \Omega_{0y}, \Omega_{0z})$ can be written as

$$\Omega_{0x} = 0,\ \Omega_{0y} = \Omega_0\sin\theta,\ \Omega_{0z} = \Omega_0\cos\theta, \tag{2.8}$$

It is assumed that the geographical φ and geomagnetic $\varphi^{'}$ latitudes coincide $\varphi = \varphi^{'}$, $\theta = \theta^{'}$ and disturbances occur in the neighborhood of the latitude $\varphi_0 = \pi/2 - \theta_0$. Further, system (2.1)–(2.5) is linearized against the background of a plane zonal shear flow (wind) $\mathbf{V}_0$: $\mathbf{V} = \mathbf{V}_0 + \mathbf{V}^{'}(x,y)$, $\rho = \rho_0 + \rho^{'}(x,y)$, $P = P_0 + P^{'}(x,y)$, where the values with a prime are the disturbed ones, while the mean (background) values have the sub-index zero (for simplicity, in the sequel we omit the prime of the perturbed values). Thus the initial system of equations for large-scale small (linear) disturbances can be written in the form

$$\begin{aligned}\frac{d\mathbf{V}_\perp}{dt} + (\mathbf{V}_\perp\bigtriangledown)\mathbf{V}_0 \quad &= \quad -\frac{\bigtriangledown P}{\rho_0} - 2\Omega\times\mathbf{V}_\perp \\ &+\frac{\sigma_\perp}{\rho_0 c^2}(B_0^2 V_\perp - B_{0y}V_y\mathbf{B}_0) + \frac{B_0\sigma_H}{\rho_0 c^2}\mathbf{V}\times\mathbf{B}_0 + \nu\triangle\mathbf{V},\end{aligned} \tag{2.9}$$

$$\gamma\frac{d\rho}{dt} + \gamma(\mathbf{V}_\perp\bigtriangledown)\rho_0 + \rho_0 div\mathbf{V}_\perp = 0, \tag{2.10}$$

$$\frac{dP}{dt} + (\mathbf{V}_\perp\bigtriangledown)P_0 + P_0 div\mathbf{V}_\perp\ = 0, \tag{2.11}$$

Here $d/dt = \partial/\partial t + \mathbf{V}_0\bigtriangledown$, $V_0(V_{0x}, 0, 0)$ is the background zonal wind velocity which, for the horizontal shear flow, is given in the form

$$\mathbf{V}_0 = ay\mathbf{e}_x \tag{2.12}$$

where a is the wind shear constant parameter, e_x is the unit vector directed along the x-axis.

In the chosen local Cartesian system for components (2.9) – (2.11) we obtain the following system of equations

$$\left(\frac{\partial}{\partial t}+ay\frac{\partial}{\partial x}\right)V_x = -\frac{1}{\rho_0}\frac{\partial P}{\partial x}-\frac{\sigma_\perp B_0^2}{\rho_0 c^2}V_x + \left(2\Omega_{0z}+\frac{\sigma_H B_0 B_{0z}}{\rho_0 c^2}-a\right)V_y+\nu\triangle V_x, \tag{2.13}$$

$$\left(\frac{\partial}{\partial t}+ay\frac{\partial}{\partial x}\right)V_y = -\frac{1}{\rho_0}\frac{\partial P}{\partial y}-\frac{\sigma_\perp B_{0z}^2}{\rho_0 c^2}V_y - \left(2\Omega_{0z}+\frac{\sigma_H B_0 B_{0z}}{\rho_0 c^2}\right)V_x+\nu\triangle V_y, \tag{2.14}$$

$$\gamma\left(\frac{\partial}{\partial t}+ay\frac{\partial}{\partial x}\right)\rho+\rho_0\left(\frac{\partial V_x}{\partial x}+\frac{\partial V_y}{\partial y}\right)=0, \tag{2.15}$$

$$\left(\frac{\partial}{\partial t}+ay\frac{\partial}{\partial x}\right)P+P_0\left(\frac{\partial V_x}{\partial x}+\frac{\partial V_y}{\partial y}\right)=0. \tag{2.16}$$

Note that in the motion equation (2.14) we have discarded the term $2\Omega_{0z}V_{0x}\rho/\rho_0$, because it is much smaller than the third term in the right-hand part of (2.14). Indeed, for the considered large-scale small disturbances $V_x/V_{0x} >> \rho/\rho_0$ [Gossard, Hooke, 1975; Pedlosky, 1984; Gill, 1986]. In this case, equation (2.15) becomes independent and defines the disturbed density of the medium when the distribution values of the velocity $V_{x,y}(x,y,t)$ are known. Thus the closed system of equations for our problem consists of three equations (2.13), (2.15) and (2.16).

To proceed with our analysis of the peculiar properties of a magnetized Rossby wave in the ionosphere, it is convenient to introduce the coordinate system with the moving axes $X_1 0_1 Y$, whose origin and 0_1 and Y_1-axis coincide with their counterparts of the equilibrium local system $X0Y$, while the X_1-axis moves together with the undisturbed (background) flow (see Fig. 1). For our problem, this is equivalent to the replacement of the variables

$$x_1 = x - ayt\ y_1 = y,\ t_1 = t, \tag{2.17}$$

or to

$$\frac{\partial}{\partial t}=\frac{\partial}{\partial t_1}-ay\frac{\partial}{\partial x_1},\ \frac{\partial}{\partial x}=\frac{\partial}{\partial x_1},\ \frac{\partial}{\partial y}=\frac{\partial}{\partial y_1}-at_1\frac{\partial}{\partial x_1}. \tag{2.18}$$

In the new variables, equations (2.13), (2.14), (2.16) take the form

$$\begin{aligned}\frac{\partial V_x}{\partial t} &= -\frac{1}{\rho_0}\frac{\partial P}{\partial x_1} - \frac{\sigma_\perp B_0^2}{\rho_0 c^2} V_x + \left(2\Omega_{0z} + \frac{\sigma_H B_0 B_{0z}}{\rho_0 c^2} - a\right) V_y \\ &+\nu\left(\frac{\partial^2}{\partial x^2} + \left(\frac{\partial}{\partial y_1} - a t_1 \frac{\partial}{\partial x_1}\right)^2\right) V_x,\end{aligned} \tag{2.19}$$

$$\begin{aligned}\frac{\partial V_y}{\partial t} &= -\frac{1}{\rho_0}\frac{\partial P}{\partial x_1} - \frac{\sigma_\perp B_0^2}{\rho_0 c^2} V_y - \left(2\Omega_{0z} + \frac{\sigma_H B_0 B_{0z}}{\rho_0 c^2}\right) V_x \\ &+\nu\left(\frac{\partial^2}{\partial x^2} + \left(\frac{\partial}{\partial y_1} - a t_1 \frac{\partial}{\partial x_1}\right)^2\right) V_y,\end{aligned} \tag{2.20}$$

$$\frac{\partial P}{\partial t} + P_0\left(\frac{\partial V_x}{\partial x_1} + \left(\frac{\partial}{\partial y_1} - a t_1 \frac{\partial}{\partial x_1}\right) V_y\right) = 0. \tag{2.21}$$

The above replacement of the variables does not mean that we have physically passed over to a new counting system, since the values V_x, V_y, P in equations (2.9)–(2.11), ((2.19)–(2.21)) are equivalent to the velocity and pressure components of wave disturbance in the Cartesian system $X0Y$. The coefficients of the initial system of linear equations (2.9)–(2.11) (or (2.13)–(2.16)) depended on the spatial coordinate y. The above mathematical transformations have changed this spatial inhomogeneity for time inhomogeneity (see equations (2.19)—(2.21)). Thus the coefficients of system (2.19)–(2.21) have become independent of the spatial variables x_1, y_1 and we are able now perform Frourier analysis of these equations with respect to the spatial variables (x_1, y_1), and consider the time evolution of these SFH separately:

$$\left\{\begin{array}{c} V_x(x_1,y_1,t_1) \\ V_y(x_1,y_1,t_1) \\ P(x_1,y_1,t_1)\end{array}\right\} = \int\!\!\int_{-\infty}^{+\infty}\!\!\int dk_{x_1} dk_{y_1} \left\{\begin{array}{c} \widetilde{V}_x(k_{x_1},k_{y_1},t_1) \\ \widetilde{V}_y(k_{x_1},k_{y_1},t_1) \\ \widetilde{P}(k_{x_1},k_{y_1},t_1)\end{array}\right\} \times \exp(ik_{x_1} + ik_{y_1} y_1), \tag{2.22}$$

here the multipliers marked by the tilde (for instance, $\widetilde{V}_x$) denote the PFH of the corresponding physical values.

In order to clarify the details of what is going on, we split the velocity of the medium into the vortical and the potential component, and introduce respectively the vorticity $\Omega = rot_z V_\perp = \partial V_y/\partial x - \partial V_x/\partial y$ and the divergence $\xi = div\, V_\perp = \partial V_x/\partial x + \partial V_y/\partial y$. Using these new functions, we can reduce the initial dynamic equations to the equations possessing a remarkable peculiarity – for large-scale processes the terms containing a time derivative have the same order as the other terms (equations of ionospheric medium motion in form (2.19), (2.20) do not have this property). Another important peculiarity of the obtained equations is that they naturally take into account the effects produced by spatial inhomogeneities of the angular velocity of the Earth's rotation Ω_0 and by the geomagnetic field $\mathbf{B}_0$. We next introduce the Rossby parameter $\beta = \partial 2\Omega_{0z}/\partial y = 2\Omega_0 \sin\theta_0/R > 0$, and also the magnetic analogies of the Rossby parameter

$$\beta_{Hz} = \frac{\partial}{\partial y}\left(\frac{\sigma_H B_0 B_{0z}}{\rho_0 c^2}\right), \ \beta_{\perp z} = \frac{\partial}{\partial y}\left(\frac{\sigma_\perp B_{0z}^2}{\rho_0 c^2}\right), \ b_{Hz} = \frac{\sigma_H B_0 B_{0z}}{\rho_0 c^2},$$

$$b_{\perp y} = \frac{\sigma_\perp B_{0y}^2}{\rho_0 c^2}, \qquad b_{\perp z} = \frac{\sigma_\perp B_{0z}^2}{\rho_0 c^2}. \tag{2.23}$$

If we substituting representation (2.22) into equations (2.19)–(2.21), omit the tilde symbol in the Fourier harmonics of the physical values and pass to the dimensionless values

$$\tau \Rightarrow 2\Omega_{0z}t_1; \qquad \Omega \Rightarrow \Omega\frac{R}{V_0}; \qquad \xi \Rightarrow \xi\frac{R}{V_0}; \qquad P \Rightarrow \frac{P}{\rho_0 V_0 \cdot 2\Omega_{0z} \cdot R};$$

$$\beta \Rightarrow \beta\frac{R}{2\Omega_{0z}}; \ \beta_{Hz} \Rightarrow \beta_{Hz}\frac{R}{2\Omega_{0z}}; \ \beta_{\perp z} \Rightarrow \beta_{\perp z}\frac{R}{2\Omega_{0z}}; \ \delta \Rightarrow \frac{P_0}{\rho_0\left(2\Omega_{0z}R\right)^2}, \tag{2.24}$$

$$b_{Hz} \Rightarrow \frac{b_{Hz}}{2\Omega_{0z}} \ b_{\perp y} \Rightarrow \frac{b_{\perp y}}{2\Omega_{0z}}; \ b_{\perp z} \Rightarrow \frac{b_{\perp z}}{2\Omega_{0z}}; \ S \Rightarrow \frac{a}{2\Omega_{0z}}; \ \nu \Rightarrow \frac{\nu}{2\Omega_{0z}R^2},$$

$$k_x = k_{x1}\cdot L; \ k_y = k_y(0) - k_x S\tau; \ k_y(0) = k_{y1}(0)R; \ k(\tau) = (k_x^2 + k_y^2(\tau))^{1/2},$$

then for each SFH we will have

$$\frac{\partial\Omega}{\partial\tau} = \left[i\frac{k_x}{k^2(\tau)}\left(\beta + \beta_{Hz}\right) - b_{\perp z} - \frac{k_y^2(\tau)}{k^2(\tau)}b_{\perp y} - \nu k^2(\tau)\right]\Omega -$$

$$-\left[l - S - i\frac{k_y(\tau)}{k^2(\tau)}\left(\beta + \beta_{Hz}\right) + b_{Hz} - \frac{k_x k_y(\tau)}{k^2(\tau)}b_{\perp y}\right]\ \xi, \tag{2.25}$$

$$\begin{aligned}\frac{\partial \xi}{\partial \tau} = & -\left[2S\frac{k_x k_x(\tau)}{k^2(\tau)} - i\frac{k_x}{k^2(\tau)}(\beta+\beta_{Hz})\right. \\ & \left.-i\frac{k_y(\tau)}{k^2(\tau)}\beta_{\perp z} + b_{\perp z} + \frac{k_x^2}{k^2(\tau)}b_{\perp y} + \nu k^2(\tau)\right]\xi \\ & +\left[1-2S\frac{k_x^2}{k^2(\tau)} - i\frac{k_y(\tau)}{k^2(\tau)}(\beta+\beta_{Hz}) + \frac{k_x}{k^2(\tau)}\beta_{\perp z} + b_{Hz}\right. \\ & \left.+\frac{k_x k_y(\tau)}{k^2(\tau)}b_{\perp y}\right]\Omega \ + k^2(\tau)P,\end{aligned} \tag{2.26}$$

$$\frac{\partial P}{\partial \tau} = -\delta\xi. \tag{2.27}$$

As seen from equations (2.25)–(2.27), the Hall conductivity (i.e. the terms with coefficients with index H) imparts the ionospheric medium an additional gyroscopicity like the Coriolis force but in the opposite direction, while the Pedersen conductivity (i.e. the terms with coefficients with index "$\perp$") intensifies the dissipative property (inductive decceleration) of the medium [Dokuchayev, 1959; Khantadze, 1973].

In the space of wave numbers, the density of total energy of wave disturbances, whose SFH are defined by formulas (2,25)—(2.27), have the form:

$$E\left[k\right] = \frac{\Omega\Omega^*}{k^2(\tau)} + \frac{\xi\xi^*}{k^2(\tau)} + \frac{PP^*}{\delta}, \tag{2.28}$$

where the asterisk denotes the complex conjugacy.

Thus, the density of total energy of wave disturbance $E\left[k\right] = E_\nu + E_c + E_e$ consists of three parts: $E_\nu = \Omega\Omega^*/k^2(\tau)$, where the first term is the energy of the vortical part of disturbances, $E_c = \xi\xi^*/k^2(\tau)$; the second term is the compressible part of the energy, $E_e = PP^*/\delta$; the third term is the elastic (potential) energy (due to disturbance elasticity), . In the absence of a shear flow ($S = 0$)and dissipative processes ($\nu = 0$, $\sigma_\perp = 0$), the total energy density of the considered wave disturbances in the ionosphere preserves its value $\partial E(\tau)/\partial\tau = 0$.

3. General analysis of the problem

In this paper we want to discuss the mistake made in describing the evolution of Rossby type waves in the presence of zonal shear flows. More specifically, it will be shown that in flows with a moderate shear, low-frequency Rossby waves, which are mostly vortical, transform – with a lapse of time – to high-frequency potential inertial waves. Actually, we mean that as a result of transformation the time scale of a wave process changes essentially. This new kind of wave transformation existing in shear flows was for the first time described in [Chagelishvili et al.,1996] for the case of magnetohydrodynamic waves. The physics of the process is simple and easy to understand by means of an example of a system of connected linear oscillators.

Let us consider two pendulums, the length of each of them changing in time (adiabatically). Such a situation makes the eigenfrequencies of these pendulums and depend on time $\omega_1(t)$ and $\omega_2(t)$. Assume that between them there exists a weak connection. Denoting the connection coefficient by $\chi(t)$ (which in the general case is also time-dependent), we can write oscillation equations of such connected pendulums in the form

$$\frac{\partial^2 X_1}{\partial t^2} + \omega_1^2(t) X_1 = \chi(t) X_2, \ \frac{\partial^2 X_2}{\partial t^2} + \omega_2^2(t) X_2 = \chi(t) X_1, \tag{3.1}$$

where X_1 and X_2 and are the oscillating variables characterizing the motion of the pendulums. If the frequencies of these pendulums differ considerably, then, irrespective of the connection, there is practically no energy exchange between them. An effective energy exchange begins as soon as the oscillator frequencies come closer to each other. The necessary conditions for an effective energy exchange are as follows [Kotkin, Serbo, 1969]:

(A) the existence of a "degeneration region", where $\left|\omega_1^2(t) - \omega_2^2(t)\right| \lesssim |\chi(t)|$;

(B) a slow passage through the "degeneration region" during a time period essentially exceeding $\chi(t)$: $|d\omega_1(t)/dt|$, $|d\omega_2(t)/dt| << |\chi(t)|$.

In other words, if at the beginning it was only the first pendulum that oscillated, after the change of its length the frequencies might come closer to each other $\omega_1(t)$ and $\omega_2(t)$ so that the conditions (A) and (B) be fulfilled. In this case, an essential (if not a greater) part of the oscillatory energy of the first pendulum is imparted to the second pendulum, which might develop strong oscillations, while the first pendulum might completely stop to oscillate. An analogous scenario might take place for Rossby type waves as well.

Indeed, the conditions (A) and (B) are valid for arbitrary oscillatory systems with connections, to which we can reduce the description of a whole number of natural physical processes. These systems are also directly applicable for analysis of the linear interaction of waves of different branches (including Rossby type waves) when their frequencies approach each other.

The evolution of each wave mode depends on the relation between four principal linear processes: (a) the drift of each SFH in the kj-space; (b) the energy exchange between a mean flow and a SFH; (c) the mutual transformation of modes; (d) disturbance energy dissipation. The process (a) is universal and takes place practically in the same manner for all wave types. The intensity of the processes (b) and (c) largely depends on a wave type and on the parameters of the system. The effectiveness of the process (d) is defined by a concrete type of dissipation.

Let us discuss each of the above processes in greater detail.

(a). From expressions (2.22), (2.24) it follows that the wave numbers of each SFH change with time along the direction normal to the background flow velocity (i.e. along the y-axis): $k_y = k_y(0) - Sk_x\tau$. Therefore each SFH drifts in the k-space in terms of linear approximation.

(b). The values of spatial characteristics (of the wave numbers $(k_x, k_y(\tau)$)) largely define the intensity of energy exchange between a SFH and the background shear flow. Therefore a linear drift brings about a change in the intensity of this exchange. However not all SFH can absorb the shear energy and gain in intensity. Only the SFH occurring in the definite region of the **k**-space (in the sequel called the intensification region, see below) increase their intensity. Each one of the harmonics keeps gaining in intensity within a limited time interval until it leaves the intensification region as a result of a linear drift. Moreover, the occurrence of SFH in this region imposes a condition on the direction (but not on the value) of their wave vector. Therefore the process of energy exchange between wave disturbances and a shear flow has an obvious anisotropic character in the **k**-space. Thus, there exist disturbances which in the linear stage of their evolution can absorb the shear flow energy only within a limited time interval and experience a temporary (transient) growth.

(c). Transformation of wave modes is a resonance process. Resonance transformation can be expected if:

- in the medium there might exist at least two wave modes;
- wave frequencies change with a lapse of time;
- the condition (a) and (b) are fulfilled.

(d). Viscous dissipation. This phenomenon becomes effective when the wave numbers grow. In the end, if some nonlinear phenomenon does not show up, then this process converts the SFH energy to heat.

It should be specially noted that in shear flows (for $S \neq 0$), because of the dependence of a wave amplitude on time a dispersion equation, which can be obtained from equations (2.25) – (2.27), is, strictly speaking, rather conditional. Nevertheless, it allows us to qualitatively understand the change of the wave frequency characteristic, and also to estimate an extent of convergence of different wave branches that takes place for certain values of $k_y(\tau)$. In the case of graphic representation of dispersion curves of Rossby and inertial waves, we always take into account the dependence on the latitudinal wave vector k_x [see, for instance, Petviashvili, Pokhotelov, 1992]. But in our case, for the clearness of the described phenomenon of wave transformation, it is convenient to consider the dependence of a frequency on k_y.

For disturbances of the plane wave type, from system (2.25)–(2.27) we obtain a conditional dispersion equation of third order for the frequency $\omega(k_x, k_y)$ (see, for instance, equation (4.7)). Solutions of this dispersion equation for different values of the shear parameter S are given in Figs. 2 and 3.

Fig. 2 shows solutions of the dispersion equation in the D-region of the atmosphere at $S = 0$. In this case there are three branches of waves (in the dimensional form):

a) branch I with frequencies ω much smaller than $2\Omega_0$ consists of Rossby waves:

$$\omega_H = -\frac{k_x V_R}{1 + k^2 r_R^2}, \tag{3.2}$$

where $V_R = \beta r_R^2$ is the Rossby velocity, $r_R = C_a/(2\Omega_{0z})$ is the Rossby radius, $C_a = (P_0/\rho_0)^{1/2}$. In the E-region the Rossby parameter β is replaced by the Rossby magnetic parameter $\beta \to \beta - \left(1/\left(\rho_0 c^2\right)\right) \partial(\sigma_H B_0 B_{0z})/\partial y$, while for the F-region we retain β;

b) branch II with frequencies $\omega \sim 2\Omega_0$ consists of inertial (gyroscopic) waves:

$$\omega_I^2 = (2\Omega_{0z})^2 \left(1 + k^2 r_R^2\right). \tag{3.3}$$

c) branch III with frequencies $\omega \gg 2\Omega_0$ consists of long acoustic waves:

$$\omega_a^2 = k^2 C_a^2. \tag{3.4}$$

For large k, inertial waves of branch II transform to long acoustic waves III running with velocity C_a.

Branch I that describes Rossby type waves practically coincides with the k_y-axis (Fig. 2), since the frequencies of these waves are much smaller than those of waves of branches II and III. It is obvious that the conditions (a) and (b) are far from being fulfilled. Thus Rossby waves are not connected with inertial waves and therefore the mutual transformation of waves does not take place for $S = 0$.

Next, let us trace the change of dispersion curves for $S \neq 0$ (see Fig. 3.). We will consider the interconnection of branches I and II, since they are the only branches whose group velocities may coincide and which may have a resonance connection. Hence it is only these waves that the mutual transformation can be expected of. When the shear value is $S = 0,8$, there exists a range of wave numbers k_x, $k_y(\tau)$, for which the low-frequency branch I and the high-frequency branch II converge and even merge.

There appears the region of degeneration (encircled by the dotted line in Fig. 3) where the conditions of wave transformation (conditions (A) and (B)) are obviously fulfilled, i.e. if it assumed that at the initial time moment only the low-frequency Rossby wave with a large value of $k_y(0)$ the vector was excited (i.e. $k_y(0)/k_x >> 1$, under this condition disturbances are practically insensitive to the presence of a shear flow), then, with a lapse of time and with a change of $k_y(\tau)$, its frequency grows, it arrives in the region of degeneration (its frequency coincides with the frequency of inertial wave II) and a certain part of its energy transforms to the energy of another wave branch (branch II). Here we obviously have a complete analogy with the interacting (connected) pendulums of variable length. Such an analogy has been discussed at the beginning of this section.

We are interested in finding out what stipulates the time dependence of wave frequencies in shear flows and what are the results of this time dependence.

The frequencies of the considered waves (for instance, (3.2), (3.3)) are the well-defined functions of the wave number $k_y(\tau)$. With a lapse of time, the change of $k_y(\tau)$ leads to a time variation of the SFH frequency – the wave "slides" along the dispersion curve of the considered modes. Hence, for certain parameter values of the system, the dispersion curves of the interacting waves in the neighborhood of the singular point ($k_y(\tau) \to 0$) converge and the wave frequencies might coincide within a limited time interval. This fact leads to the resonance of waves and the mutual transformation of their energy even in the case of a small time variation of wave frequencies. A typical picture of the evolution of the considered process is shown in Fig. 4.

Let us assume that a wave harmonic of lower branch II (inertial wave) with the wave number $k_y(0) = 0,4$ occurred initially at point 1. Because of the variation of $k_y(\tau)$ with a lapse of time, the wave slides along the dispersion curve ($1 \to 2 \to 3$) and its frequency varies. In the neighborhood of point 3, there also lies a part of the dispersion curve of upper branch I (Rossby wave), i.e. on the dispersion curve there appears the region of degeneration. Thus the frequency of the upper branch at point 4 very closely approaches the frequency of the lower branch at point 3 (both frequencies may even coincide). Now it becomes possible for the resonance interaction – transformation of waves (points $3 \to 4$) to take place. The transformed wave (i.e. the wave of the upper branch) continues to slide along the upper dispersion curve (along points $4 \to 5 \to 6$).

Thus, the appearance of the degeneration region on the dispersion curve brings about an energy exchange between wave disturbance SFH in shear flows and the sliding of wave SFH along the dispersion curve make the process obvious.

The basis of energy exchange between wave disturbances and a shear flow is the so-called lift-up mechanism [Landahl,1975; Chagelishvili et al., 1996], when disturbances transfer fluid from the regions with a higher flow velocity to the ones with a lower velocity, and vice versa. The energy exchange between SFH and a mean flow is the more intensive, the higher is the velocity with which a disturbed fluid element moves along the shear or, in other words, the larger is the projection of the velocity of perturbation SFH along the shear (in our case, along the Y-axis). Note that the values of this velocity essentially differ for SFH of incompressible and compressible wave disturbances.

For SFH of incompressible waves we have the relation $\mathbf{k} \perp \mathbf{V}$. So, for $k_y(0) >> k_x$, i.e. when the wave vector is practically directed along the Y-axis (along the shear), the SFH velocity is almost normal to this direction. As a result, the velocity projection along the shear is small and, by the lift-up mechanism, there is practically no energy exchange between SFH and the mean flow. A small energy exchange for these SFH may occur only in a limited time interval when $k_y(\tau) \leq k_x$ (a temporary (transient) growth of disturbances).

The situation is radically different for SFH of compressible waves. In this case, the angle between $\mathbf{k}$ and $\mathbf{V}$ differs considerably from $\pi/2$. Moreover, $\mathbf{V}$ is nearly parallel to $\mathbf{k}$. It is obvious that for this a direction of the velocity $\mathbf{V}$, the energy exchange between SFH and the background flow may take place even for $k_y(\tau) >> k_x$ (see Section 4).

Thus we see that the flow of a fluid element along the shear cannot uniquely provide an energy exchange between SFH of wave disturbances and the mean flow. According to [Landahl,1975; Chagelishvili et al., 1996], the energy exchange between the mean flow and a wave may take place if, besides the flow along the shear, the waves also produce a disturbance of thermal pressure. Hence it can be assumed that SFH of ionospheric Rossby type waves and inertial waves, which exchange their energy with the mean flow for certain values of $k_y(\tau)$, also produce disturbances of thermal pressure (see Section 4, Figs. 5 and 12).

4. Results of the numerical solution

In order to trace the evolution of SFH of a magnetized Rossby wave and an inertial wave in ionospheric shear flows (in zonal winds), we performed the numerical solution of equations (2.25)–(2.28). We solved the initial Cauchy problem for a system consisting of three linear ordinary differential equations of first order with complex coefficients. More exactly, we solved the following system of six equations but with real coefficients:

$$\frac{\partial \Omega_1}{\partial \tau} = -a_2\Omega_1 - a_1\Omega_2 - a_3\xi_1 - a_4\xi_2, \tag{4.1}$$

$$\frac{\partial \Omega_2}{\partial \tau} = -a_2\Omega_2 - a_1\Omega_1 - a_3\xi_2 - a_4\xi_1, \tag{4.2}$$

$$\frac{\partial \xi_1}{\partial \tau} = -b_1\xi_1 - b_2\xi_2 + b_3\Omega_1 + b_4\Omega_2 + k^2(\tau)P_1, \tag{4.3}$$

$$\frac{\partial \xi_1}{\partial \tau} = -b_1\xi_2 - b_2\xi_1 + b_3\Omega_2 + b_4\Omega_1 + k^2(\tau)P_2, \tag{4.4}$$

$$\frac{\partial P_1}{\partial \tau} = -\delta\xi_1, \tag{4.5}$$

$$\frac{\partial P_2}{\partial \tau} = \delta\xi_2. \tag{4.6}$$

Here we introduced new variables $\Omega = \Omega_1 + i\Omega_2$, $\xi = \xi_1 + i\xi_2$, $P = P_1 + iP_2$, i is the imaginary unity, while the real coefficients $a_1, a_2, ..., b_1, b_2, ..., \delta$ are related to the coefficients of equations (2.25)–(2.27) and have different values for different ionospheric layers (D, E, F). The expressions for them will be given below.

Calculations were performed for different values of the parameters of the medium and wave disturbances. Analysis of the numerical solution showed the energy exchange between various wave branches as well as between waves and the background flow.

4.1. A choice of initial physical values.

To single out an individual kind of waves in the initial state and in the pure form, the physical values of waves were chosen under the assumption that, initially, only a certain (Rossby type or inertial) wave without any noticeable admixtures of other modes was excited.

Thus, the initial data for the physical values contained in equations (2.25)–(2.28) can be chosen from these equations provided that $k_y(0) >> k_x$ and, accordingly, $S \approx 0$. Indeed, for $|k_y(\tau)/k_x| >> 1$, in the formula for the meridianal wave number $k_y(\tau) = k_y(0) - k_x S \cdot r$ can be assumed that $k_y(\tau) \approx k_y(0)$ during a moderate time interval, $S\tau \lesssim 1$. Note that the choice of a value $k_y(0)/k_x >> 1$ as an initial one does not restrict the variation region of the parameter $k_y(\tau)/k_x$, since, with a lapse of time, first $|k_y(\tau)/k_x|$ monotonically drops to zero and then grows and takes all real values. Therefore we can neglect the influence of the shear flow on the initial distribution of physical values in the system, i.e. for the initial time moment it can be assumed $S \to 0$ in the right-hand parts of equations (2.25)—(2.27) that In this case, in system (2.25)–(2.27) all coefficients are constant and the initial physical values can be defined by the representation $\partial A(\tau)/\partial\tau \approx -i\omega A(\tau)$, where ω is the initial excitement frequency. Now, system (2.25)–(2.27) or, which is the same, system (4.1)– (4.6), transforms to a homogeneous system of six algebraic equations for six unknowns (for the real and the imaginary part of the physical values: $\Omega^0 = \Omega_1^0 + i\Omega_2^0$, $\xi^0 = \xi_1^0 + i\xi_2^0$, $P^0 = P_1^0 + iP_2^0$). Hence expressions for the initial physical values Ω_1^0,Ω_2^0,...,P_1^0,P_2^0 (whose explicit expressions are not given here because of their inconvenience) includes as a parameter the frequency of the considered wave disturbance $\omega^{I,II} = \omega_1^{I,II} + \omega_2^{I,II}$. Further, we choose for ω^I or ω^{II} the results of the corresponding numerical solution of the third order conditional dispersion equation obtained from equations (2.25)—(2.28) (see also Section .3)

$$\omega^3 + [a_1 + b_2 + i\,(b_1 + a_2)]\,\omega^2 +$$

$$+\left[a_1b_2 + a_4b_4 - a_2b_1 - a_3b_3 - \delta k^2 + +i\,(a_1b_1 + a_2b_2 + a_3b_4 + a_4b_3)\right]\omega -$$

$$-\delta k^2\,(a_1 + ia_2) = 0. \tag{4.7}$$

Substituting the corresponding root of equation (4.7) into the expressions for Ω_1^0,Ω_2^0,...,P_1^0,P_2^0 and taking into account that $S \neq 0$, we can obtain the initial excitement of an individual mode of the magnetized Rossby wave or of the inertial wave. Using these initial data and the numerical results obtained for equations (4.1) – (4.6), we can trace the evolution of the initially singled out (excited) wave disturbance in the dissipative ionosphere.

In addition to the physical values, the initial equations (4.1)–(4.6) include as coefficients the parameters characterizing the equilibrium state of the medium. Since throughout the ionospheric thickness, the equilibrium parameter values vary in a wide range, the wave disturbance characteristics will accordingly be essentially different for different layers (D, E, F). Therefore it is advisable to give the coefficient values of the initial equations (4.1)–(4.7) for different layers of the ionosphere.

D- **layer**.The characteristic values of this layer which is up to 80 km high satisfy the relations $\nu_{en} >> \nu_{ei}$, $\nu_{in}\nu_{ei} >> \omega_{Be}\omega_{Bi}$ and $\omega_{Be} >> \nu_{en}$. Moreover, by virtue of relations (2.5) we see that the terms with the coefficients σ_H and $\sigma_\perp$ in the right-hand parts of equations (4.1)–(4.6) are much smaller than the terms with β and Ω_{0z}, and the corresponding coefficients are defined by the expressions

$$a_1 = \frac{k_x}{k^2(\tau)}\beta,\ a_2 = \nu k^2(\tau),\ a_3 = 1 - S,\ a_4 = \frac{k_y(\tau)}{k^2(\tau)}\beta,$$

$$b_1 = \nu k^2(\tau) + 2S\frac{k_x k_y(\tau)}{k^2(\tau)},\ b_2 = \frac{k_x}{k^2(\tau)}\beta,\ b_3 = 1 - 2S\frac{k_x^2}{k^2(\tau)},\ b_4 = \frac{k_y(\tau)}{k^2(\tau)}\beta. \tag{4.8}$$

Thus in the initial equations (4.1)–(4.7) there remain only the terms characterizing the neutral atmosphere and, accordingly, they describe the evolution of the usual Rossby wave (3.2), the inertial wave (3.3) and the long gravitation wave (3.4).

E**- layer**. For the ionospheric E-layer, whose height varies from 80 to 150 km, it can be assumed that $\nu_e \approx \nu_{en}$, $\omega_{Be}\omega_{Bi} >> \nu_{in}\nu_{en}$; $\nu_{in}^2 >> \omega_{Bi}^2$. In this case the Hall conductivity is $\sigma_H \approx eN/B_0$ and prevails over the transverse conductivity $\sigma_H >> \sigma_\perp \approx \sigma_H\omega_{Bi}/\nu_{in}$. In the respective equations (4.1)–(4.6) the terms with σ_H become of the same order as the terms with coefficients Ω_{0z}. Thus, for the E-layer, the coefficients in the right-hand parts of equations (4.1) – (4.6) take the form:

$$a_1 = \frac{k_x}{k^2(\tau)}\beta_{Hz},\ a_2 = \nu k^2(\tau),\ a_3 = l - S - b_{Hz},\ a_4 = \frac{k_y(\tau)}{k^2(\tau)}\beta_{Hz};$$

$$b_1 = \nu k^2(\tau) + 2S\frac{k_x k_y(\tau)}{k^2(\tau)},\ b_2 = \frac{k_x}{k^2(\tau)}\beta_{Hz}, \tag{4.9}$$

$$b_3 = 1 - 2S\frac{k_x^2}{k^2(\tau)} - b_{Hz},\ b_4 = \frac{k_y(\tau)}{k^2(\tau)}\beta_{hz},\ b_{Hz} = \frac{N}{N_n}\frac{\omega_{ie}}{\Omega_{0z}}\cos\theta_0,$$

$$\beta_{Hz} = \beta - \frac{N}{N_n}\frac{\omega_{ie}}{\Omega_{0z}}\cos\theta_0,\ \omega_{ie} = \frac{eB_0}{M}.$$

The influence of the equilibrium geomagnetic field is described by the parameters β_{Hz}, b_{Hz} and is produced by the presence of Hall currents in the ionospheric E-region.

F**- layer**. Throughout the F-layer (150 to 500 km high) the following relations are fulfilled: $\omega_{Be}\omega_{Bi} >> \nu_e\nu_{en}$ and $\omega_{Bi} >> \nu_{in}$. According to (2.5), the F-layer has the prevailing transverse conductivity $\sigma_H/\sigma_\perp \approx (M\omega_{Bi} - m\omega_{Be})/(m\nu_e) \to 0$. Therefore for the coefficients a, b and β we have

$$a_1 = \frac{k_x}{k^2(\tau)}\beta,\ a_2 = \nu k^2(\tau) + \frac{k_y(\tau)}{k^2(\tau)}b_{\perp y} + b_{\perp z},$$

$$a_3 = l - S - \frac{k_x k_y(\tau)}{k^2(\tau)} b_{\perp y},\ a_4 = \frac{k_y(\tau)}{k^2(\tau)}\beta$$

$$b_1 = \nu k^2(\tau) + 2S\frac{k_x k_y(\tau)}{k^2(\tau)} + \frac{k_x^2}{k^2(\tau)} b_{\perp y} + b_{\perp z},\ b_2 = \frac{1}{k^2(\tau)}(k_x\beta + k_y\beta_{\perp z}),$$

$$b_3 = 1 - 2S\frac{k_x^2}{k^2(\tau)} + \frac{k_x k_y(\tau)}{k^2(\tau)} b_{\perp y},\ b_4 = \frac{1}{k^2(\tau)}(k_y(\tau)\beta - k_x\beta_{\perp z}),$$

$$\beta_{\perp z} = \frac{N}{N_n}\frac{\nu_{in}}{\Omega_{0z}}\frac{2\sin 2\theta_0}{(1+3\cos^2\theta_0)^2} \tag{4.10}$$

$$b_{\perp y} = \frac{N}{N_n}\frac{\nu_{in}}{\Omega_{0z}}\frac{\sin^2\theta_0}{2(1+3\cos^2\theta_0)},\ b_{\perp z} = \frac{N}{N_n}\frac{\nu_{in}}{\Omega_{0z}}\frac{2\cos^2\theta_0}{1+3\cos^2\theta_0}.$$

The presence of the equilibrium inhomogeneous geomagnetic field in the medium is reflected in expressions for the parameters $b_{\perp y,z}$, $\beta_{\perp z}$, while the interaction of this field with the medium is due to Pedersen currents.

4.2. Energy exchange with the background flow and the transformation of a magnetized Rossby wave to inertial waves

We begin the analysis of numerical experiments by considering the case of excitation of Rossby type waves in the ionospheric D-region.

Intensification. At the initial time moment, only the low-frequency planetary Rossby wave with a large value of the meridianal wave vector $k_y(0)$, $k_y(0)/k_x = 50 >> 1$ and $\beta = 0.1$, $S = 0.8$, $\delta = 1$, $\nu = 0$, $k_x = 2$, $k_y(0) = 100$, $P_1^0 = 1$ was excited. Some of the results of numerical solution of equations (4.1)–(4.6) and (2.28) are presented in Figs. 5—11. When $k_y(0)/k_x >> 1$, the Rossby wave is mainly vortical (see Fig. 9, the vortex energy has the largest share in the total energy) and practically incompressible (at the initial evolution stage, the compressible and the elastic part of the excitement energy are equal to zero, see Figs. 10 and 11). As mentioned at the end of Section 3, in the incompressible stage, wave excitations may absorb the background flow energy only if $k_y(\tau) \approx k_x$. Indeed, as seen from Figs. 5—8, due to the linear drift, $k_y(\tau)$ decrease with a lapse of time, but for $k_y(\tau) >> k_x$ the energy exchange between the background flow and the SFH of the Rossby wave is not essential. For times when already $k_y(\tau) \approx k_x$ the SFH of the Rossby wave actively absorbs the shear energy and gains in intensity (Fig. 8), i.e. the SFH is now in the region of intensification (see also Section 3). The SFH intensification stops at the time moment $k_y(\tau^*) = 0$ (see Fig. 8 for $\tau = \tau^* = k_y(\tau^*)/(Sk_x) \approx$

62.5). Then, for $k_y(0)/k_x < 0$ or in a time interval $\tau^* < \tau \leq \tau_1$, it begins to give back some part of the energy to the mean flow.

It should be noted that irrespective of the fact that in an initial time interval $0 \leq \tau < \tau^*$, a Rossby type wave is always present the medium necessarily (see Fig. 9), while in Figs. 5—7 its presence is practically unnoticeable (an almost straight line in the figures) because of its large time scale (as compared with an inertial wave).

Fig. 5 shows that in ionospheric shear flows, a Rossby type wave causes a strong excitation of the thermal pressure of the P medium and, according to Section 3, there occurs an intensive energy exchange between the background flow and the wave.

Transformation. As seen from Figs. 8 and 9, with the evolution of the initial excitement, the share of the vortex component in the total energy keeps decreasing until it becomes negligibly small (for $\tau_2\tilde{}80$) and a greater part of the Rossby wave energy is pumped into the energy of inertial waves. Thus the Rossby wave transforms to inertial waves. (The total energy (Fig. 8) and the SFH (Figs. 5—7) now have high-frequency oscillations). Thus, $\tau = 0$ the energy is concentrated in vortical low-frequency modes (Rossby waves), for $\tau >> \tau^*$ the whole energy is concentrated in potential high-frequency disturbances, i.e. in inertial waves (Figs. 10 and 11). Transformation of Rossby type waves to inertial ones starts from the moment $\tau = \tau^*$ and goes on within a limited time interval in which the conditions (A) and (B) (see Section 3) are fulfilled and these two branches get interconnected. It should be noted that the waves of branches I and II are connected not only with each other, but (as mentioned above) also with the mean flow and thus they exchange their energy with the latter flow. A greater part of the Rossby wave energy undergoes transformation. It can be said that by the time moment $\tau = \tau_1$(see Fig. 8) only the (inertial) wave of branch II remain in the flow. With a lapse of time, the latter wave intensifies by absorbing the shear energy (see the part of the curve for $\tau > \tau_1$ in Fig. 8). Figs. 8—11 clearly demonstrate how much the evolution process of a Rossby wave changes because of the transformation of this wave to an inertial wave: if the latter process had not taken place, then the Rossby energy would have weakened according to the law marked by the dotted line in Fig. 8 and given its energy to the background flow.

From the numerical results it also follows that at the levels of the E-region, where $\beta_{Hz} = 0$ (at an altitude of 115 km in the daytime and 150 km at night) [Aburjania et al., 2003], planetary Rossby type waves practically do not get excited, but at these altitudes inertial waves might make their appearance.

As the parameter b_{Hz} increases (for $\beta_{Hz} \neq 0$), the interaction of Hall currents with the ionospheric E-layer and the geomagnetic field $\mathbf{B}_0$ becomes essential and the evolution dynamics of the initially excited magnetized Rossby wave changes noticeably. Wave excitations first absorb the shear energy and the whole SFH disturbance energy keeps increasing until the time moment $\tau = \tau^* = k_y(\tau^*)/(Sk_x)$. Further, disturbances (for $\tau > \tau^*$, $k_y(0)/k_x < 0$) give back their energy to the background flow (Fig. 12). During this process, the initially excited Rossby type wave (with $k_y(0)/k_x = 50 >> 1$) begins with a lapse of time (when $k_y(\tau) \approx k_x$) to transform to an inertial wave and keeps giving the latter wave the bulk of its energy until the time moment $\tau^* = 6.25$ (see Fig. 13). It should be specially noted that the reverse process takes place with a lapse of time (when $\tau > \tau^* = 62.5$): the inertial wave returns the bulk of its energy to the Rossby type wave (Figs. 13 and 14). Finally, for $\tau \geq \tau^*$ and $k_y(0)/k_x < 0$, the combined inertial wave and Rossby type wave return their energy to the medium even in the absence of dissipative processes (see Figs. 12–14, where $\nu = 0$, and compare with Figs. 8–10). Here, too, the energy exchange with the shear flow occurs by the lift-up mechanism (see Section 3).

In the ionospheric F-region, the excitation dynamics of a magnetized Rossby wave and its further evolution depend on the interaction of Pedersen currents with the medium and the geomagnetic field. This interaction eventually reduces to the inductive damping of wave disturbances. As the parameter $\beta_{\perp z}$ increases (at small values of the parameters $b_{\perp y}$, $b_{\perp z} << 1$), the evolution of the initially excited Rossby type wave (generation, intensification, and transformation to inertial waves) qualitatively occurs in the same manner as in the D-region with the only difference that in this case the interaction with the background flow is more effective and, accordingly, the SFH disturbance amplitude has a noticeably higher value. As the parameters $b_{\perp y}$, $b_{\perp z}$ increase, the inductive damping intensity of disturbances (Rossby type and inertial waves) grows and the wave energy transforms to heat (see Fig. 15).

4.3. Interaction of inertial waves with the background flow and the transformation to a Rossby type wave.

If in the D-region of the ionosphere it is only the inertial wave (3.3) that gets initially excited, then the evolution of the corresponding SFH in the shear flow differs radically from the above-discussed case of Rossby type waves.

In Figs. 16—21 we present some of the results of numerical solution of equations (4.1)–(4.6) and (2.28), when at the initial time moment only the inertial wave gets excited for $\beta = 0.1$, $S = 0.8$, $\delta = 1$, $\nu = 0$, $k_x = 2$, $k_y(0) = 100$, $P_1^0 = 1$. As already mentioned in Section 3, an inertial (potential) wave belongs to the class of compressible wave and therefore it has an intensive energy exchange with the background flow for an arbitrary relation between $k_y(0)$ and k_x (as different from a Rossby wave which noticeably interacts with the background flow only at $k_y(0)\tilde{}k_x$).

We see that with a lapse of time the amplitudes (as well as their frequencies) of inertial wave SFHs decrease (for $\tau = \tau^* = k_y(\tau^*)/(Sk_x) \approx 62.5$): some part of the SFH energy is transferred to the background flow (Figs. 16—18). The meridianal wave vector also decreases with time, $k_y(\tau^*) \to 0$ and, in the neighborhood of the time $\tau^* = k_y(\tau^*)/(Sk_x) \approx 62.5$ the frequency of inertial waves approaches that of the Rossby type wave and the degeneration region is formed (see Section 3), where a part of the energy of the initial inertial wave (Fig. 19) (approximately 20% as seen from Figs. 19—21) transforms to the energy of the Rossby type wave (Fig. 20). Thus the generation of a Rossby type wave takes place (Fig. 20). Further, the SFH arrives in the intensification region, where $k_y(0)/k_x < 0$ (for $\tau > \tau^*$) and the combination of the inertial wave and the Rossby wave begins to absorb the background flow energy: we observe the growth of SFH amplitudes P_1, Ω_1, ξ_1, their frequencies (Figs. 16—18) and the SFH energy (Figs. 19 and 21).

If the inertial wave is initially excited in the E-region of the ionosphere, its further evolution occurs in a somewhat different manner. Because of the Hall conductivity, the inertial wave first transfers a part of its energy to the background flow, while a part of its energy simultaneously transforms to the Rossby type wave energy. The transformation of waves takes place when disturbance SFH arrive in the degeneration domain at $\tau = \tau^* = k_y(0)/k_x$. Next, for $\tau > \tau^*$, the combined inertial wave and magnetized Rossby wave absorb the shear energy and gain in intensity. Finally, for $\tau >> \tau^*$, the wave mixture returns the energy to the background flow even in the absence of dissipative processes ($\nu = 0$). As the parameter b_{Hz} (i.e. the Hall current amplitude) increases, the effectiveness of generation of a magnetized Rossby wave grows.

In the F-region, the evolution of the initial inertial wave is mainly condioned by the Pedersen conductivity (which bring about an intensive damping of disturbances, see, for instance, Fig. 15) and qualitatively coincides with the evolution of a magnetized Rossby wave as described at the end of Subsection 4.3.

4.4. Damping of large-scale wave disturbances in the shear flow

As has already been mentioned, in the shear flow we observe the SFH drift in the space of wave numbers. Thus, with a lapse of time, the radial component of the SFH wave vector increases, $k_y(\tau) = k_y(0) - Sk_x\tau$, i.e. the disturbance length decreases along the meridian (for $\tau \to \infty$, $l_y = 2\pi/|k_y(\tau)| \to 0$). Usually, in a solid medium the subdivision of scales takes place at the expense of nonlinear processes [Zaslavski, Sagdeev, 1988]. However in our case a monotone decrease of disturbance scales occurs in the linear regime. For short-wave disturbances, the influence of dissipative processes (viscosity in our case) is essential (see Fig. 22, starting with $\tau > 120$). Due to dissipation, the disturbance energy is transferred in the form of heat to the medium and, eventually, a practically complete damping of wave disturbances takes place (see Fig. 22, at $\tau \approx 300$)

This process can be schematically described in the plane k_xOk_y (see Fig. 23). Here we will consider only the plane $k_x > 0$, since the results can analogously be applied to the plane $k_x < 0$. Without taking nonlinear processes into account, the dynamics of the considered (Rossby and inertial) wave disturbances is defined by the following basic processes: 1) a SFH drift in the **k**-space; 2) the drawing of the background flow energy by SFH; 3) mutual transformation of modes; 4) viscous and inductive dampings. Each of these processes takes place for different values of the wave vector **k**. Therefore, for a clear understanding and analysis of what is going on, the region of evolution of these processes in the **k**-space can be considered differentially. Let us assume that dissipation becomes essential for SFH with a wave number satisfying the inequality $|\mathbf{k}| > k_\nu$ (in Fig. 23 this region is drawn by vertical lines to the outer part of the half-plane with radius $|\mathbf{k}| = k_\nu$), where k_ν depends on a concrete type of dissipation. We also assume that the energy exchange between the shear flow and wave disturbances takes place in the region drawn by horizontal and sloping lines in Fig. 23 (this is the region of intensification and transformation). Disturbances like Rossby waves or inertial waves with an arbitrary point **k** can always be generated in the ionospheric medium due to thermal fluctuation.

Let us discuss the evolution path of a SFH which at the initial time moment is at point 1 in Fig 23. The wave number changes with time along the Y-axis of this harmonic $k_y(\tau)$, which leads to its drift along the direction marked by the arrows. As soon as at a certain moment of time τ the harmonic reaches point 2, its energy begins to grow (at the expense of the shear energy) and keeps growing until it transforms to another wave branch (point 3 in Fig. 23). Further, the combined initial and transformed waves go on absorbing the shear energy and gain in intensity (the region hatched by sloping lines). Then the drifting SFH reaches point 4, where dissipative processes become active and transform the SFH energy to heat. Other Fourier harmonics which correspond to other points of the **k**-space evolve analogously. After the Fourier harmonic leaves point 1, this point does not remain vacant, since because of thermal effects new fluctuations occupy this points and evolve in an analogous manner.

Therefore the pumping of shear flow energy to the wave perturbation energy and the mutual transformation of modes followed by their dissipation in the medium are permanent processes, which may to a strong heating of the medium. It is obvious that the heating intensity depends on the initial disturbance level and the shear flow parameter S.

5. Conclusion

In this paper we investigate the linear stage of the evolution of SFH of a magnetized Rossby wave and inertial wave disturbances in the dissipative ionosphere in the presence of a shear flow (smooth-inhomogeneous zonal wind). Based on the numerical solution and theoretical analysis of the corresponding system of dynamic equations, new mechanisms are found, which account for the pumping of shear flow energy to wave disturbance energy, an extremal intensification (by several orders) of waves, the mutual transformation of eigenmodes and the conversion of perturbation energy to heat.

The intensification of a magnetized Rossby wave and an inertial wave may take place for certain values of the parameters of the medium, shear and waves. This makes an unusual way of shear flow heating in the ionosphere: waves draw up the shear flow energy and pump it through the mutual linear transformation and linear drift of SFH in the space of wave numbers (subdivision of disturbance scales) to the damping domain. Finally, the viscosity and inductive damping convert this pumped energy to heat. The process is permanent and may lead to a strong heating of the medium. The heating intensity depends on the initial disturbance level and shear flow parameters.

A remarkable feature of a shear flow is the diminution of wave disturbance scales in the linear stage, which is due to a linear drift of disturbance SFH in the space of wave numbers and, accordingly, to the pumping of energy to the dissipation region (with short scales).

The intensification of wave disturbance SFH and the mutual transformation of modes take place within a limited time interval (transiently) as long as the corresponding conditions of intensification and a sufficiently strong interconnection of modes are fulfilled.

The mutual transformation of eigenmodes (of Rossby and inertial waves) may take place even in the spatial-homogeneous ionosphere ($\rho_0 = const$), when the background wind velocity is inhomogeneous. We should emphasize the fact that this transformation mechanism was revealed in the framework of nonmodal mathematical analysis (these processes were not taken into account in the case of a more traditional modal approach). Thus the nonmodal approach that takes into account the non-orthogonality of eigenfunctions of linear wave dynamics problems, has proved to be a more adequate mathematical language for investigating wave processes in shear flows.

The character of the wave transformation mechanism considered in this paper is essentially different from the previously known linear mechanism of wave transformation in the inhomogeneous plasma [Erokhin, Moiseev, 1973]. The transformation of waves in the case of medium density inhomogeneities takes place in a limited space (across the density inhomogeneity) as long as this inhomogeneity exists, while in our case the transformation of linear waves occurs throughout the shear flow volume but in a limited time interval (transiently). It is obvious that for this phenomenon to take place it is necessary that at least two wave modes exist in the medium. The realization of the considered wave transformation mechanism is possible if the conditions (A) and (B) given in Section 3 are fulfilled.

The effect of the revealed mutual transformation of Rossby type waves and inertial waves in the ionosphere with an inhomogeneous zonal wind makes us revise some notions existing in dynamic meteorology and in the models of general circulation of the atmosphere, ocean, ionosphere and magnetosphere with the participation of Rossby type planetary waves. This especially concerns the interpretation of experimental and observation data, when it is necessary to take into account a possibility of mutual transformation of waves with different time and spatial scales in shear flows.

Thus, the use of the Charney-Obukhov equation or the vortex transfer equation (where the averaging is performed over high frequency values) as a mathematical model describing the dynamics of large-scale Rossby type waves in the atmosphere or in the ocean is, mildly speaking, unjustified, since shear flows always exist in the atmosphere and in the ocean. A more adequate alternative is the mathematical model that takes into account the fact that in the ionospheric medium there exist not only Rossby type waves but also other wave modes differing essentially in a time scale from Rossby waves.

The presence of the electromagnetic ponderomotive force, i.e. of an inhomogeneous geomagnetic field, Hall and Pedersen currents in different ionospheric layers increases the effectiveness of interaction and energy exchange between wave disturbances and the background shear flow.

Finally, note that our analysis is performed in the case of a homogeneous flow shear (assuming a linear dependence of the velocity on a coordinate). However, the results will mainly be the same for an inhomogeneous (non-linear) profile of the background shear velocity if the characteristic transverse length of the wave l_y is less than the characteristic length of the nonlinear velocity profile L_y, $l_y << L_y$, or if the background wind profile is approximated by the linear term in a Taylor series [Volponi et al.,2001].

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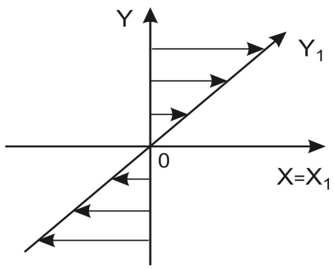

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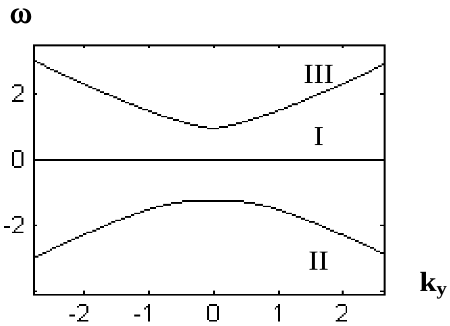
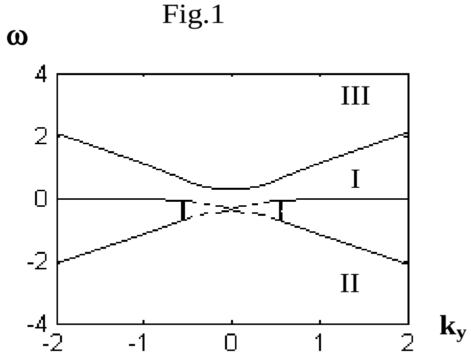


Fig.2



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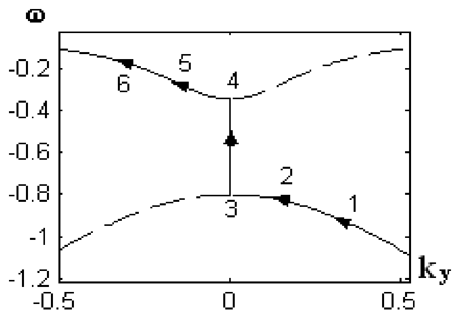

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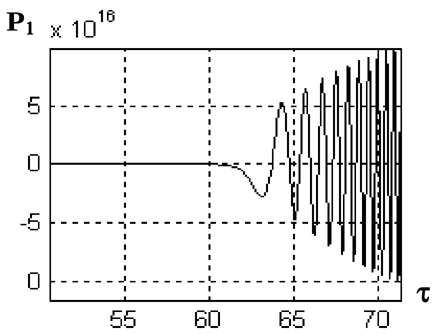
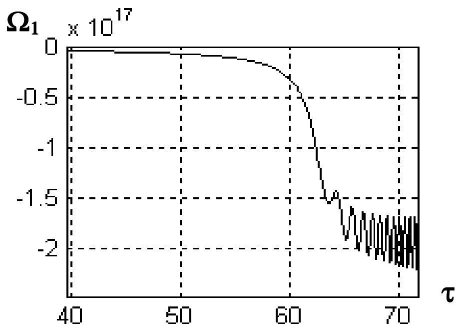


Fig. 5



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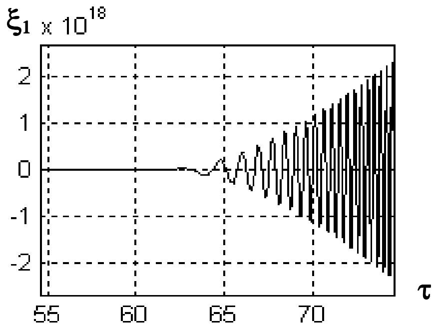

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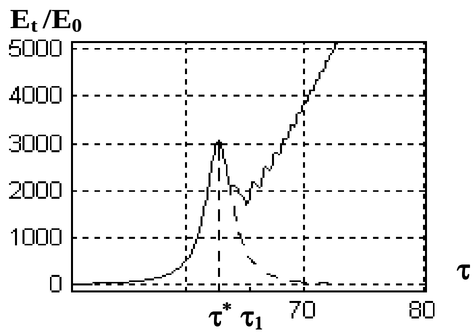


Fig.8

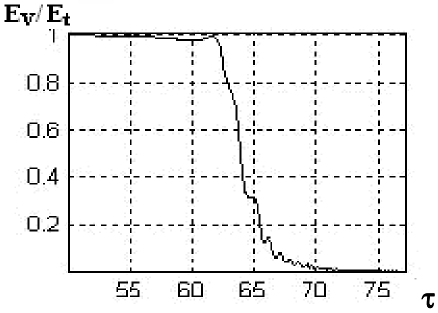


Fig.9

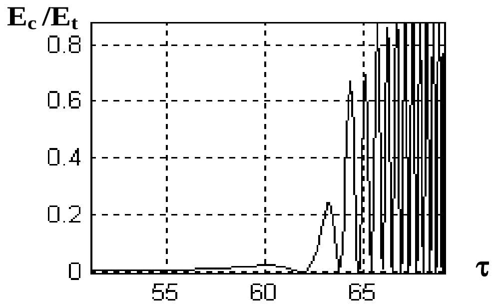
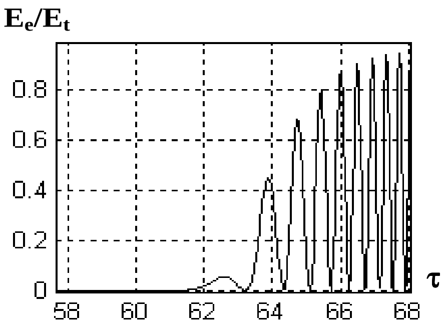


Fig.10



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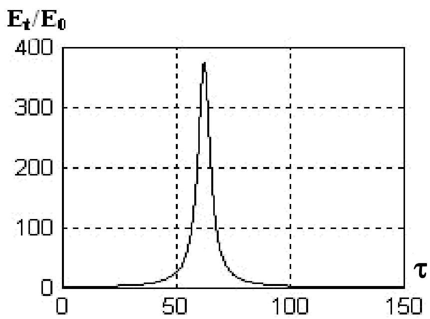

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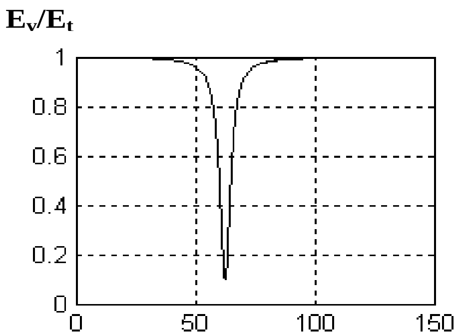

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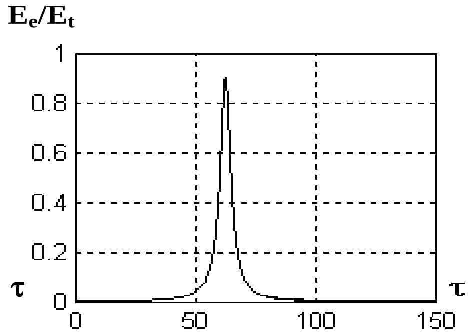

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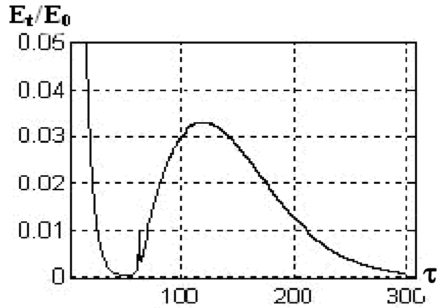

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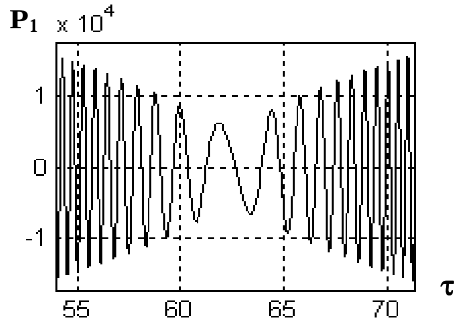

Fig. 16

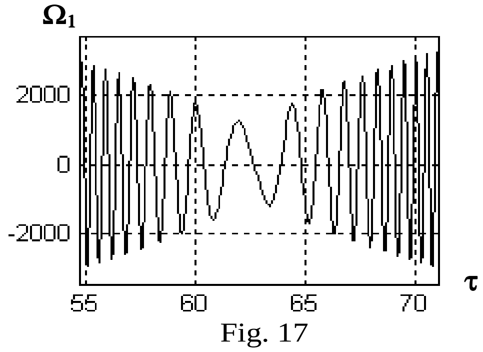

Fig. 17

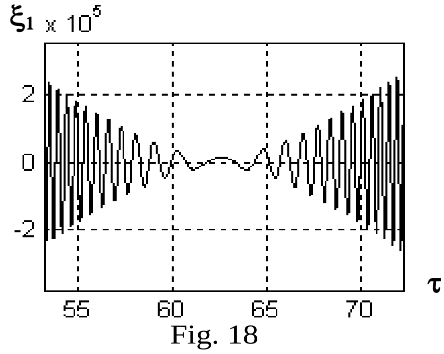

Fig. 18

Fig.1. The local Cartesian system with X_1OY_1-axes and the system with moving -axes. The arrows show the directions of the background flow velocity $V_{0x} = ay$. The X_1-axis moves together with the shear flow.

Fig.2. Dispersion curves for $\beta = 0.1$, $S = 0$, $\delta = 1$, $\nu = 10^{-7}$, $k_x = 0.5$, $k_y(0) = 10$, $P_1^0 = 1$.

Fig.3. Dispersion curves for $\beta = 0.1$, $S = 0.8$, $\delta = 1$, $\nu = 10^{-7}$, $k_x = 0.5$, $k_y(0) = 10$, $P_1^0 = 1$

Fig.4. A typical picture of wave transformation $\beta = 0.15$, $S = 0.42$, $\delta = 1.8$, $\nu = 10^{-6}$, $k_x = 0.4$, $k_y(0) = 1$, $P_1^0 = 1$.

Fig.5. The time evolution of SFH $P_1 = \Re P$ for the parameters $\beta = 0.1$, $S = 0.8$, $\delta = 1$, $\nu = 0$, $k_x = 2$, $k_y(0) = 100$, $P_1^0 = 1$ in the D-region, when only a Rossby type wave is excited at the initial moment of time.

Fig.6. The time evolution of SFH $\Omega_1 = \mathbf{Re}\Omega$ for the parameters $\beta = 0.1$, $S = 0.8$, $\delta = 1$, $\nu = 0$, $k_x = 2$, $k_y(0) = 100$, $P_1^0 = 1$ in the$D$-region, when only a Rossby type wave is excited at the initial moment of time.

Fig.7. The time evolution of SFH $\xi_1 = \mathbf{Re}\xi$ for the parameters $\beta = 0.1$, $S = 0.8$, $\delta = 1$, $\nu = 0$, $k_x = 2$, $k_y(0) = 100$, $P_1^0 = 1$ in the D-region, when only a Rossby type wave is excited at the initial moment of time.

Fig.8. The time dependence of the total SFH E_t/E_0 energy for $\beta = 0.1$, $S = 0.8$, $\delta = 1$, $\nu = 0$, $k_x = 2$, $k_y(0) = 100$, $P_1^0 = 1$ in the D-region, when only a Rossby type wave is excited at the initial moment of time ($E_0 = E_t(\tau = 0)$).

Fig.9. The time dependence of a ratio of the vortical energy part E_ν to the total energy E_t for $\beta = 0.1$, $S = 0.8$, $\delta = 1$, $\nu = 0$, $k_x = 2$, $k_y(0) = 100$, $P_1^0 = 1$ in the D-region, when only a Rossby type wave is excited at the initial moment of time.

Fig.10. The time dependence of a ratio of the compressible energy part E_c to the total energy E_t for $\beta = 0.1$, $S = 0.8$, $\delta = 1$, $\nu = 0$, $k_x = 2$, $k_y(0) = 100$, $P_1^0 = 1$ in the D-region, when only a Rossby type wave is excited at the initial moment of time.

Fig.11. The time dependence of a ratio of the elastic energy part E_e to the total energy E_t for $\beta = 0.1$, $S = 0.8$, $\delta = 1$, $\nu = 0$, $k_x = 2$, $k_y(0) = 100$, $P_1^0 = 1$ in the D-region, when only a Rossby type wave is excited at the initial moment of time.

Fig.12. The time evolution of total SFH E_t/E_0 energy for $\beta = 0.1$, $S = 0.8$, $\delta = 1$, $\nu = 0$, $k_x = 2$, $k_y(0) = 100$, $P_1^0 = 1$, $b_H = 5$ in the E- region, when only a magnetized Rossby wave is excited at the initial moment of time.

Fig.13. The time dependence of a ratio of the vortical energy part E_v to the total energy E_t for $\beta = 0.1$, $S = 0.8$, $\delta = 1$, $\nu = 0$, $k_x = 2$, $k_y(0) = 100$, $P_1^0 = 1$, $b_H = 5$ in the E- region, when only a magnetized Rossby wave is excited at the initial moment of time.

Fig.14. The time dependence of a ratio of the elastic energy part E_e to the total energy E_t for $\beta = 0.1$, $S = 0.8$, $\delta = 1$, $\nu = 0$, $k_x = 2$, $k_y(0) = 100$, $P_1^0 = 1$, $b_H = 5$ in the E-region, when only a magnetized Rossby wave is excited at the initial moment of time

Fig.15. The time evolution of the total SFH energy E_t/E_0 for $\beta = 0.1$, $S = 0.8$, $\delta \approx 2$, $\nu = 0$, $k_x = 2$, $k_y(0) = 100$, $P_1^0 = 1$, $\beta_{1z} = 5$, $b_{1y} = 0.01$, $b_{1z} = 0.01$ in F- region, when only a magnetized Rossby wave is excited at the initial moment of time.

Fig.16. The time evolution of SFH $P_1 = \mathbf{Re}P$ for $\beta = 0.1$, $S = 0.8$, $\delta \approx 1$, $\nu = 0$, $k_x = 2$, $k_y(0) = 100$, $P_1^0 = 1$, the parameters in the D- region, when only an inertial wave is excited at the initial moment of time.

Fig.17. The time evolution of SFH $\Omega_1 = \mathbf{Re}\Omega$ for $\beta = 0.1$, $S = 0.8$, $\delta = 1$, $\nu = 0$, $k_x = 2$, $k_y(0) = 100$, $P_1^0 = 1$ the parameters in the D- region, when only an inertial wave is excited at the initial moment of time.

Fig.18. The time evolution of SFH $\xi_1 = \mathbf{Re}\xi$ for $\beta = 0.1$, $S = 0.6$, $\delta = 0.3$, $\nu = 0$, $k_x = 2.5$, $k_y(0) = 50$, $P_1^0 = 1$ the parameters in the D-region, when only an inertial wave is excited at the initial moment of time.

Fig.19. The time dependence of the logarithm of the total SFH energy E_t for $\beta = 0.1$, $S = 0.6$, $\delta = 0.3$, $\nu = 0$, $k_x = 2.5$, $k_y(0) = 50$, $P_1^0 = 1$ in the D- region, when only an inertial wave is excited at the initial moment of time.

Fig.20. The time dependence of the logarithm of the vortical energy part E_v for $\beta = 0.1$, $S = 0.8$, $\delta = 1$, $\nu = 0$, $k_x = 2$, $k_y(0) = 100$, $P_1^0 = 1$ in the D- region, when only an inertial wave is excited at the initial moment of time.

Fig.21. The time dependence of the logarithm of the elastic energy part E_e for $\beta = 0.1$, $S = 0.8$, $\delta = 1$, $\nu = 0$, $k_x = 2$, $k_y(0) = 100$, $P_1^0 = 1$ in the D- region, when only an inertial wave is excited at the initial moment of time.

Fig.22. The damping of the total SFH energy E_t/E_0 with a lapse of time for $\beta = 0.1$, $S = 0.8$, $\delta \approx 1$, $\nu = 10^{-6}$, $k_x = 2$, $k_y(0) = 100$, $P_1^0 = 1$, $b_H = 5$ in the - region, when only the magnetized Rossby wave is excited at the initial moment of time.

Fig.23. A qualitative representation of the evolution of wave disturbances in the k_xOk_y-plane. In the region marked by horizontal and sloping lines we observe the intensification and mutual transformation of wave disturbances (Figs. 8 and 19). In the external region marked by vertical lines the energy of disturbance transforms to heat due to dissipative processes in the medium.