A STUDY OF DISLOCATION IN THERMOELASTICITY OF DIPOLAR BODIES WITH VOIDS

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Abstract

The aim of our study is do derive a relation of Knopoff-De Hoop type for displacement vector fields within context of thermoelastic dipolar bodies with voids. Then, as a consequence, an explicit expression of the body loadings equivalent to the dislocation, is obtained.

Key words and phrases: dipolar body, voids, dislocation.
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1. Introduction

The theory of thermo-microstretch elastic solids was first elaborated by Eringen, [4]. In short, this is a theory of elasticity with microstructure that include intrinsic rotations and microstructural expansion and contractions.

This theory can be useful in the applications which deal with porous materials, such as geological materials, solid packed granular materials and many others.

On the other hand, materials which operate at elevated temperatures will invariably be subjected to heat flow at some time during normal use. Such heat flow will involve a non-linear temperature distribution which will inevitable give rise to thermal stresses. For these reasons, the development, design and selection of materials for high temperature applications requires a great deal of care. The role of the pertinent material properties and other variables which can affect the magnitude of thermal stress must be considered.

The present paper must be considered as a first step to a better understanding of microstretch and thermal stress in the study of above enumerated materials.

The reciprocity and representation relations that appear in our study constitute a powerful theoretical tools in the assessment of the theory of seismic-sources mechanism, in the studies connected with seismic wave propagation.

Also, we think that this paper is a good help to understanding the application of microstretch mechanism to earthquake problems.

2. Basic equations

For convenience, the notations and terminology chosen are almost identical to those of our studies [6], [7]. Our paper is concerned with an anisotropic and homogeneous material.

Let the body occupy, at time $t = 0$, a properly regular region B of the three-dimensional Euclidian space, bounded by the piece-wise smooth surface ∂B and we denote the closure of B by $\bar{B}$. We refer the motion of the body to a fix system of rectangular Cartesian axes $Ox_i, i = 1, 2, 3$ and adopt the Cartesian tensor notation. Points in B are denoted by x_j and $t\in[0, \infty)$ is temporal variable. Throughout this work the Einstein summation convention over repeated indices is used. The subscript j after comma indicates partial differentiation with respect to the spatial argument x_j. All Latin subscripts are understood to range over the integers $(1, 2, 3)$, while the Greek indices have the range $(1, 2)$. A superposed dot denotes the derivatives with respect to the $t-$ time variable. Also, the spatial argument and the time argument of a function will be omitted when there is no likelihood of confusion.

For clarity and simplification in presentation, the regularity hypotheses on the considered functions will be omitted.

On these grounds, the field equations in the dynamic theory of thermoelasticity of dipolar bodies with voids are, (see, [6], [7]):

- the equations of motion

$$\begin{aligned} \left(\tau_{ij} + \eta_{ij}\right)_{,j} + \varrho F_1 &= \varrho \ddot{u}_i, \\ \mu_{ijk,i} + \eta_{jk} + \varrho G_{jk} &= I_{kr} \ddot{\varphi}_{jr}; \end{aligned} \tag{1}$$

- the balance of the equilibrated forces

$$\lambda_{i,i} - s + \varrho L = \varrho k \ddot{\sigma}; \tag{2}$$

- the energy equation.

$$\varrho T_0 \dot{\eta} = q_{i,i} + \varrho r; \tag{3}$$

- the constitutive equations

$$\begin{aligned}
\tau_{ij} &= C_{ijmn}\varepsilon_{mn} + G_{mnij}\gamma_{mn} + F_{mnrij}\chi_{mnr} \\
&\quad + a_{ij}\sigma + d_{ijk}\sigma_{,k} - \alpha_{ij}\theta, \\
\eta_{ij} &= G_{ijmn}\varepsilon_{mn} + B_{ijmn}\gamma_{mn} + D_{ijmnr}\chi_{mnr} \\
&\quad + b_{ij}\sigma + e_{ijk}\sigma_{,k} - \beta_{ij}\theta, \\
\mu_{ijk} &= F_{ijkmn}\varepsilon_{mn} + D_{mnijk}\gamma_{mn} + A_{ijkmnr}\chi_{mnr} \\
&\quad + e_{ijk}\sigma + f_{ijkm}\sigma_{,m} - \omega_{ijk}\theta, \\
h_i &= d_{mni}\varepsilon_{mn} + e_{mni}\gamma_{mn} + f_{mnri}\chi_{mnr} \\
&\quad + d_i\sigma - a_i\theta + P_{ij}\sigma_{,j}, \\
s &= -a_{ij}\varepsilon_{ij} - b_{ij}\gamma_{ij} - e_{ijk}\chi_{ijk} - \xi\sigma - d_i\theta_{,i} + m\theta, \\
S &= S_0 + \alpha_{ij}\varepsilon_{ij} + \beta_{ij}\gamma_{ij} + \omega_{ijk}\chi_{ijk} + m\sigma + a_i\sigma_{,i} + \alpha\theta, \\
q_i &= k_{ij}\theta_{,j},
\end{aligned} \tag{4}$$

-the kinetic relations

$$\varepsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right),\ \gamma_{ij} = u_{j,i} - \varphi_{ij}$$

$$\chi_{ijk} = \varphi_{jk,i},\ \theta = T - T_0,\ \sigma = \varphi - \varphi_0 \tag{5}$$

In the above equations we have used the following notations:

ϱ - the constant reference mass density, $S-$ the specific entropy, T_0 - the constant absolute temperature of the body in its reference state, I_{ij} - the components of microinertia, k - the equilibrated inertia, u_i - the components of displacement vector, φ_{ij} - the components of dipolar displacement tensor, φ - the volume distribution function which in the reference state is φ_0, σ - the change in volume fraction measured from the reference state, $\varepsilon_{ij},\ \gamma_{ij},\ \chi_{ijk}$ - the kinematic characteristic of the strain, $\tau_{ij},\ \eta_{ij},\ \mu_{ijk}$ - the components of the stress tensors, λ_i - the components of the equilibrated stress vector, q_i - the components of the heat conduction vector, F_i - the components of the body forces, G_{ij} - the components of the dipolar body forces, L - the extrinsic equilibrated body force.
Finally, the tensors $C_{ijmn},\ B_{ijmn},\ ...,k_{ij}$ represent the characteristic functions of the material and they obey the symmetry relations:

$$C_{ijmn} = C_{mnij} = C_{jimn},\ B_{ijmn} = B_{mnij},$$

$$A_{ijkmnr} = A_{mnrijk},\ F_{ijkmn} = F_{ijknm},\ G_{ijmn} = G_{ijnm}, \tag{6}$$

$$a_{ij} = a_{ji},\ d_{ijk} = d_{jik},\ P_{ij} = P_{ji},\ k_{ij} = k_{ji}.$$

The entropy inequality implies

$$k_{ij}\theta_{,i}\theta_{,j} \geq 0 \tag{7}$$

To the system of field equations (1)-(5) we adjoin the following initial conditions

$$\begin{aligned} u_i(x,0) = u_i^0(x),\ \dot{u}_i(x,0) = u_i^1(x),\ x \in \bar{B} \\ \varphi_{jk}(x,0) = \varphi_{jk}^0(x),\ \dot{\varphi}_{jk}(x,0) = \varphi_{jk}^1(x),\ x \in \bar{B} \\ \theta(x,0) = \theta^0(x),\ \sigma(x,0) = \sigma^0(x),\ \dot{\sigma}(x,0) = \sigma^1(x),\ x \in \bar{B} \end{aligned} \tag{8}$$

and the following prescribed boundary conditions

$$\begin{aligned} u_i = \bar{u}_i \ on\ \partial B_1 \times [0,t_0),\ (\tau_{ij} + \eta_{ij})\, n_j = \bar{t}_i \ on\ \partial B_1^c \times [0,t_0), \\ \varphi_{jk} = \bar{\varphi}_{jk} \ on\ \partial B_2 \times [0,t_0),\ \mu_{ijk} n_k = \bar{\mu}_{jk} n_k \ on\ \partial B_2^c \times [0,t_0), \\ \sigma = \bar{\sigma} \ on\ \partial B_3 \times [0,t_0),\ \lambda_i n_i = \bar{h} \ on\ \partial B_3^c \times [0,t_0), \\ \theta = \bar{\theta} \ on\ \partial B_4 \times [0,t_0),\ q_i n_i = \bar{q} \ on\ \partial B_4^c \times [0,t_0), \end{aligned} \tag{9}$$

where $\partial B_1, \partial B_2, \partial B_3$ and ∂B_4 with respective complements $\partial B_1^c, \partial B_2^c, \partial B_3^c$ and ∂B_4^c are subsets of ∂B such that

$$\partial B_1 \cup \partial B_1^c = \partial B_2 \cup \partial B_2^c \partial B_3 \cup \partial B_3^c = \partial B_4 \cup \partial B_4^c = \partial B,$$

$$\partial B_1 \cap \partial B_1^c = \partial B_2 \cap \partial B_2^c \partial B_3 \cap \partial B_3^c = \partial B_4 \cap \partial B_4^c = \emptyset,$$

n_i are the components of the unit outward normal to ∂B, t_0 is some instant that may be infinite.

Finally, The quantities u_i^0, u_i^1, φ_{jk}^0, φ_{jk}^1, θ^0, σ^0, σ^1, $\bar{u}_i$, $\bar{t}_i$, $\bar{\varphi}_{jk}$, $\bar{\mu}_{jk}$, $\bar{\sigma}$, $\bar{\theta}$, $\bar{h}$ and $\bar{q}$ are prescribed functions in their domains.

By a solution of the mixed initial boundary value problem of the thermoelasticity theory of dipolar bodies with voids in the cylinder $\Omega_0 = B \times [0, t_0)$ we mean an ordered array $(u_i,\ \varphi_{jk},\ \sigma,\ \theta($ which satisfies the equations (1), (2) and (3) for all $(x,t) \in \Omega_0$, the boundary conditions (9) and the initial conditions (8).

3. Main results

Let u and v be functions defined on $\bar{B} \times [0,\infty)$ and continuous on $[0,\infty)$ with respect to t for each $x \in \bar{B}$. We denote by $u * v$ the convolution of u and v, that is

$$(u * v)(x,t) = \int_0^t u(x, t-\tau) v(x,\tau) d\tau.$$

Let us introduce the notations

$$
\begin{aligned}
g(t) &= t,\ h(t)=1, \\
f_i &= \varrho g * F_i + \varrho\left[t u_i^1(x) + u_i^0(x)\right], \\
g_{jk} &= \varrho g * G_{jk} + I_{kr}\left[t\varphi_{jr}^1(x) + \varphi_{jr}^0(x)\right], \\
l &= \varrho g * L + \varrho k\left[t\sigma^1(x) + \sigma^0(x)\right], \\
w &= \varrho h * \tau + \varrho T_0 S_0.
\end{aligned}
\tag{10}
$$

Following the same procedure used by Iesan in [9], it is easy to prove the following result, that enables us to give an alternative formulation of the initial boundary value problem in which the initial data are incorporated into the field of equations.

Theorem 1. *The functions u_i, φ_{jk}, σ, θ, τ_{ij}, η_{ij}, μ_{ijk} and q_i satisfy the equations (1), (2), (3) and the initial conditions (8) if and only if they satisfy the following system of equations*

$$
\begin{aligned}
& g * (\tau_{ji} + \eta_{ji})_{,j} + f_i = \varrho u_i \\
& g * (\mu_{ijk,i} + \eta_{jk}) + g_{jk} = I_{kr}\varphi_{jr} \\
& g * (\lambda_{i,i} - s) + l = \varrho k \sigma \\
& g * q_{i,i} + w = \varrho k T_0 S
\end{aligned}
\tag{11}
$$

In our following estimations, we will use the formulation (12) of the mixed problem. We wish to find the behavior of the considered medium when embedded in B there is a discontinuity surface Σ for the displacements, dipolar displacements, the stretch and the temperature. The sides of Σ are denoted by Σ^- and Σ^+.

Let v_i be the components of the unit normal vector of Σ^-directed from $-$ to $+$ side. Then on Σ we have the conditions

$$
\begin{aligned}
u_i^+ - u_i^- &= U_i,\ \left(\tau_{ji}^+ + \eta_{ji}^+\right) v_j = \left(\tau_{ji}^- + \eta_{ji}^-\right) v_j \\
\varphi_{jk}^+ - \varphi_{jk}^- &= \Phi_{jk},\ \mu_{ijk}^+ v_k = \mu_{ijk}^- v_k \\
\sigma^+ - \sigma^- &= \Psi,\ \lambda_j^+ v_j = \lambda_j^- v_j \\
\theta^+ - \theta^- &= \emptyset,\ q_j^+ v_j = q_j^- v_j
\end{aligned}
\tag{12}
$$

where f^+ and f^- are the limits of the function $f(x)$ as x approaches a point on the side $+$ or $-$ of the surface Σ, respectively, and U_i, Φ_{ij}, Ψ and $\emptyset$ are prescribed functions. In this way we can consider the equations (12) in the domain $B \setminus \Sigma$.

Let us consider two different systems of loadings for the body

$$\begin{aligned} G^{(\alpha)} &= \left\{F_i^{(\alpha)},\, G_{jk}^{(\alpha)},\, L^{(\alpha)},\, r^{(\alpha)},\, \bar{u}_i^{(\alpha)},\, \bar{\varphi}_{ij}^{(\alpha)},\, \bar{\sigma}_i^{(\alpha)},\right. \\ &\quad \left.\bar{\theta}_i^{(\alpha)},\, \bar{t}_i^{(\alpha)},\, \bar{\mu}_{jk}^{(\alpha)},\, \bar{h}^{(\alpha)},\, \bar{q}^{(\alpha)},\, U_i,\, \Phi_{ij},\, \Psi,\, \emptyset\right\}, \\ \alpha &= 1,2 \end{aligned}$$

and two corresponding solutions

$$\begin{aligned} S^{(\alpha)} &= \left\{u_i^{(\alpha)},\, \varphi_{jk}^{(\alpha)},\, \sigma^{(\alpha)},\, \theta^{(\alpha)},\, \varepsilon_{ij}^{(\alpha)},\, \gamma_{ij}^{(\alpha)},\right. \\ &\quad \left.\tau_{ij}^{(\alpha)},\, \eta_{ij}^{(\alpha)},\, chi_{ijk}^{(\alpha)},\, \lambda_i^{(\alpha)},\, s^{(\alpha)},\, q_i^{(\alpha)}\right\}, \\ \alpha &= 1,2 \end{aligned}$$

For the sake of simplicity, we now introduce the notations

$$\begin{aligned} t_i &= (\tau_{ij} + \eta_{ij})\, n_j,\ T_i = \left(\tau_{ij}^{+} + \eta_{ij}^{+}\right) v_j \\ \mu_{jk} &= \mu_{ijk} n_j,\ M_{jk} = \mu_{ijk}^{+} v_i \\ \lambda &= \lambda_i n_i,\ \Lambda = \lambda_i^{+} v_i \\ q &= q_i n_i,\ Q = q_i^{+} v_i \end{aligned} \tag{13}$$

In the following theorem, we prove a reciprocity relation of Betti type.

Theorem 2. *If a dipolar thermoelastic body with voids is subjected to two system of loadings $G^{(\alpha)}$ then between the corresponding solutions $S^{(\alpha)}$ there is the following reciprocity relation*

$$\begin{aligned} & \int_B \left(f_i^{(1)} * u_i^{(2)} + g_{jk}^{(1)} * \varphi_{jk}^{(2)} + l^{(1)} * \sigma^{(2)}\right. \\ & \left. -\frac{1}{T_0} g * w^{(1)} * \theta^{(2)}\right) dV \\ & + \int_{\partial B} g * \left(t_i^{(1)} * u_i^{(2)} + \mu_{jk}^{(1)} * \varphi_{jk}^{(2)}\right. \\ & \left. +\lambda^{(1)} * \sigma^{(2)} - \frac{1}{T_0} h * q^{(1)} * \theta^{(2)}\right) dA \\ & - \int_{\partial B} \left(T_i^{(1)} * U_i^{(2)} + M_{jk}^{(1)} * \Phi_{jk}^{(2)} + \Lambda^{(1)} * \Psi^{(2)}\right. \\ & \left. -\frac{1}{T_0} h * Q^{(1)} * \Theta^{(2)}\right) dA \\ = & \int_B \left(f_i^{(2)} * u_i^{(1)} + g_{jk}^{(2)} * \varphi_{jk}^{(1)} + l^{(2)} * \sigma^{(1)}\right. \\ & \left. -\frac{1}{T_0} g * w^{(2)} * \theta^{(1)}\right) dV \end{aligned}$$

$$
\begin{aligned}
&+\int_{\partial B} g * \Big(t_i^{(2)} * u_i^{(1)} + \mu_{jk}^{(2)} * \varphi_{jk}^{(1)} \\
&+\lambda^{(2)} * \sigma^{(1)} - \frac{1}{T_0} h * q^{(2)} * \theta^{(1)}\Big)\, dA \\
&-\int_{\partial B} g * \Big(T_i^{(2)} * U_i^{(1)} + M_{jk}^{(2)} * \Phi_{jk}^{(1)} \\
&+\Lambda^{(2)} * \Psi^{(1)} - \frac{1}{T_0} h * Q^{(2)} * \Theta^{(1)}\Big)\, dA.
\end{aligned}
$$

Proof. In view of symmetry relations (6) and with the aid of the constitutive relations (4), by direct calculations it is easy to obtain

$$
\begin{aligned}
& \tau_{ij}^{(1)} * \varepsilon_{ij}^{(2)} + \eta_{ij}^{(1)} * \gamma_{ijk}^{(2)} + \mu_{ijk}^{(1)} * \chi_{ij}^{(2)} \\
& +\lambda_i^{(1)} * \sigma_{,i}^{(2)} + s^{(1)} * \sigma^{(2)} - \varrho\theta^{(1)} * S^{(2)} \\
= \; & \tau_{ij}^{(2)} * \varepsilon_{ij}^{(1)} + \eta_{ij}^{(2)} * \gamma_{ijk}^{(1)} + \mu_{ijk}^{(2)} * \chi_{ij}^{(1)} \\
& +\lambda_i^{(2)} * \sigma_{,i}^{(1)} + s^{(2)} * \sigma^{(1)} - \varrho\theta^{(2)} * S^{(1)}
\end{aligned}
\tag{14}
$$

Let us introduce the notation

$$
\begin{aligned}
I_{\alpha\beta} \;=\; & \int_B g * \Big[\tau_{ij}^{(\alpha)} * \varepsilon_{ij}^{(\beta)} + \eta_{ij}^{(\alpha)} * \gamma_{ij}^{(\beta)} + \mu_{ijk}^{(\alpha)} * \chi_{ijk}^{(\beta)} \\
& +\lambda_i^{(\alpha)} * \sigma_{,i}^{(\beta)} + s^{(\alpha)} * \sigma^{(\beta)} - \varrho\theta^{(\alpha)} * S^{(\beta)}\Big]\, dV
\end{aligned}
\tag{15}
$$

Based on the identity (16) and the notation (17), it is easy to see that

$$I_{\alpha\beta} = I_{\beta\alpha}$$

With the aid of the equations of motion and the equation (12), we can write

$$
\begin{aligned}
& g * \Big[\tau_{ij}^{(\alpha)} * \varepsilon_{ij}^{(\beta)} + \eta_{ij}^{(\alpha)} * \gamma_{ijk}^{(\beta)} + \mu_{ijk}^{(\alpha)} * \chi_{ij}^{(\beta)} \\
& +\lambda_i^{(\alpha)} * \sigma_{,i}^{(\beta)} + s^{(\alpha)} * \sigma^{(\beta)} - \varrho\theta^{(\alpha)} * S^{(\beta)}\Big] \\
= \; & g * \Big[\Big(\tau_{ji}^{(\alpha)} + \eta_{ji}^{(\alpha)}\Big) * u_j^{(\beta)} + \mu_{ijk}^{(\alpha)} * \varphi_{ijk}^{(\beta)} \\
& +\lambda_i^{(\alpha)} * \sigma^{(\beta)} - \frac{1}{T_0} h * q_i^{(\alpha)} * \theta^{(\beta)}\Big]_{,i} \\
& +f_i^{(\alpha)} * u_i^{(\beta)} + g_{jk}^{(\alpha)} * \varphi_{jk}^{(\beta)} + l^{(\alpha)} * \sigma^{(\beta)} - \frac{1}{T_0} g * w^{(\alpha)} * \theta^{(\beta)} \\
& -\Big[\varrho u_i^{(\alpha)} * u_i^{(\beta)} + I_{kr}\varphi_{jr}^{(\alpha)} * \varphi_{jk}^{(\beta)} + \varrho k \sigma^{(\alpha)} * \sigma^{(\beta)}\Big] \\
& +\frac{1}{T_0} g * h * k_{ij}\theta_{,i}^{(\alpha)} * \theta_{,i}^{(\beta)}.
\end{aligned}
\tag{16}
$$

By integrating in (19) and using the divergence theorem, we are led to

$$\begin{aligned} I_{\alpha\beta} \quad = \quad & \int_B \left(f_i^{(\alpha)} * u_i^{(\beta)} + g_{jk}^{(\alpha)} * \varphi_{jk}^{(\beta)} + l^{(\alpha)} * \sigma^{(\beta)} \right. \\ & \left. - \frac{1}{T_0} g * w^{(\alpha)} * \theta^{(\beta)} \right) dV \\ & + \int_{\partial B} g * \left(t_i^{(\alpha)} * u_i^{(\beta)} + \mu_{jk}^{(\alpha)} * \varphi_{jk}^{(\beta)} \right. \\ & \left. + \lambda^{(\alpha)} * \sigma^{(\beta)} - \frac{1}{T_0} h * q^{(\alpha)} * \theta^{(\beta)} \right) dA \\ & - \int_\Sigma g * \left(T_i^{(\alpha)} * U_i^{(\beta)} + M_{jk}^{(\alpha)} * \Psi_{jk}^{(\beta)} \right. \\ & \left. + \Lambda^{(\alpha)} * \emptyset^{(\beta)} - \frac{1}{T_0} h * Q^{(\alpha)} * \Theta^{(\beta)} \right) dA. \end{aligned} \tag{17}$$

Finally, introducing (20) into (17), we arrive at the desired result (15).
It is easy to see that at the absence of the discontinuities we obtain the generalization, in the context of the thermoelasticity of dipolar bodies with voids, of the previous results established in the classical thermoelastodynamics.

Based on the relation (15), we now calculate the thermomechanical body loadings equivalent to a given dislocation. To this aim, we assume that $u_i^{(2)}$, $\varphi_{jk}^{(2)}$, $\sigma^{(2)}$ and $\theta^{(2)}$, as functions of (t, x), where $x = (x_i)$, are of class $C^\infty(B \times [0, \infty))$. Of course, if the functions $u_i^{(2)}$, $\varphi_{jk}^{(2)}$, $\sigma^{(2)}$ and $\theta^{(2)}$ are given, then by means of the equations (12), we can determine the functions $F_i^{(2)}$, $G_{jk}^{(2)}$, $L^{(2)}$ and $r^{(2)}$. We restrict our considerations to the case when $U_i^{(2)} = \Phi_{jk}^{(2)} = \Psi^{(2)} = \Theta^{(2)}$ and $S^{(2)}$ correspond to the faulted medium. Then, in the case of identical boundary conditions, we obtain

$$\begin{aligned} & \int_B \varrho \left(F_i^{(1)} * u_i^{(2)} + G_{jk}^{(1)} * \varphi_{jk}^{(2)} \right. \\ & \left. + L^{(1)} * \sigma^{(2)} - \frac{1}{T_0} h * r^{(1)} * \theta^{(2)} \right) dV \\ = \quad & \int_B \varrho \left(F_i^{(2)} * u_i^{(1)} + G_{jk}^{(2)} * \varphi_{jk}^{(1)} \right. \\ & \left. + L^{(2)} * \sigma^{(1)} - \frac{1}{T_0} h * r^{(2)} * \theta^{(1)} \right) dV \\ & - \int_\Sigma g * \left(T_i^{(2)} * U_i^{(1)} + M_{jk}^{(2)} * \Phi_{jk}^{(1)} \right. \\ & \left. + \Lambda^{(2)} * \Psi^{(1)} - \frac{1}{T_0} h * Q^{(2)} * \Theta^{(1)} \right) dA. \end{aligned} \tag{18}$$

In view of (14), we have

$$\begin{aligned}
T_i^{(2)} &= [(C_{ijmn}+G_{ijmn})\,\varepsilon_{ij}+(G_{mnij}+B_{ijmn})\,\gamma_{ij}\\
&\quad +(F_{mnrij}+D_{ijmnr})\,\chi_{ij}+(a_{ij}+b_{ij})\,\sigma_{ij}\\
&\quad +(d_{ijk}+e_{ijk})\,\sigma_{,k}-(\alpha_{ij}+\beta_{ij})\,\theta]\,v_j,\\
T_{jk}^{(2)} &= [F_{ijkmn}\varepsilon_{mn}+D_{mnijk}\gamma_{mn}+A_{ijkmnr}\chi_{mnr}\\
&\quad +e_{ijk}\sigma+f_{mijk}\sigma_m-\omega_{ijk}\theta]\,v_i,\\
\Lambda^{(2)} &= [d_{mni}\varepsilon_{mn}+e_{mni}\gamma_{mn}+f_{mnri}\chi_{mnr}+d_i\sigma-a_i\theta+P_{ij}\sigma_{,j}]\,v_i,\\
\Theta^{(2)} &= k_{ij}\theta_{,i}v_j.
\end{aligned}$$

Taking into account the definition of the Dirac translated measure, δ, we can prove the relation of the following type

$$\begin{aligned}
\psi_i(\xi,t) &= \int_B \psi_i(x,t)\delta(x-\xi)dV,\\
\psi_{i,j}(\xi,t) &= \int_B \psi_i(x,t)\delta_{,j}(x-\xi)dV
\end{aligned} \tag{19}$$

and then the relation (19) can be rewritten as follows

$$\begin{aligned}
&\int_B \varrho\left[\left(F_i^{(1)}+\mathcal{F}_i\right)*u_i^{(2)}+\left(G_{jk}^{(1)}+\mathcal{G}_{jk}\right)*\varphi_{jk}^{(2)}\right.\\
&\left.+\left(L^{(1)}+\mathcal{L}\right)*\sigma^{(2)}-\frac{1}{T_0}*\left(r^{(1)}+\mathcal{R}\right)*\theta^{(2)}\right]dV\\
&\int_B \varrho\left(F_i^{(2)}*u_i^{(1)}+G_{jk}^{(2)}*\varphi_{jk}^{(1)}+L^{(2)}*\sigma^{(1)}-\frac{1}{T_0}h*r^{(2)}*\theta^{(1)}\right)dV.
\end{aligned} \tag{20}$$

In the above relations we have used the notations

$$\begin{aligned}
\mathcal{F}_k &= -\frac{1}{\varrho}\int_\Sigma\Big[(C_{jirk}+2G_{jirk}+B_{jirk})\,U_i^{(1)}\\
&\quad +(F_{jimrk}+D_{rkjim})\,\Phi_{im}^{(1)}+(d_{rji}+e_{rji})\,\Psi_i^{(1)}\Big]\,\delta_{,r}(x-\xi)v_jdA_\xi\\
\mathcal{G}_{lk} &= -\frac{1}{\varrho}\int_\Sigma\Big\{[(G_{lkji}+B_{jilk})\,\delta(x-\xi)-(F_{rjilk}+D_{rlkji})\,\delta_{,r}(x-\xi)]\,U_i^{(1)}\\
&\quad +[D_{mjilk}\delta(x-\xi)-A_{rjimlk}\delta_{,r}(x-\xi)]\,\Phi_{im}^{(1)}\\
&\quad +[e_{jlk}\delta(x-\xi)-f_{rjlk}\delta_{,r}(x-\xi)]\,\Psi^{(1)}\Big\}\,v_jdA_\xi\\
\mathcal{L} &= -\frac{1}{\varrho}\int_\Sigma\Big\{[(a_{ji}+b_{ji})\,\delta(x-\xi)-(d_{rji}+e_{rji})\,\delta_{,r}(x-\xi)]\,U_i^{(1)}\\
&\quad +[c_{jim}\delta(x-\xi)-f_{rjim}\delta_{,r}(x-\xi)]\,\Phi_{im}^{(1)}\\
&\quad +[d_j\delta(x-\xi)-P_{rj}\delta_{,r}(x-\xi)]\,\Psi^{(1)}\Big\}\,v_jdA_\xi
\end{aligned}$$

$$
\begin{aligned}
\mathcal{R} \ = \ & \frac{1}{\varrho}\int_{\Sigma} T_0 \left[(\alpha_{ji}+\beta_{ji})\,\dot{U}_i^{(1)}+\omega_{jim}\dot{\Phi}_{im}^{(1)}+a_j\dot{\Psi}^{(1)}\right]\delta(x-\xi)- \\
& -k_{jr}\Theta^{(1)}\delta_{,r}(x-\xi)\Big\}\, v_j dA_{\xi}.
\end{aligned}
$$

Taking into account the relation (23) we deduce that the effect of the discontinuities across the surface Σ can be represented by extra external body loads and heat supply.

Although these are supposed to act in an unfaulted medium and cannot in any sense represent real forces acting in the real medium, they may nevertheless provide, as pointed in the papers [1] and [3], a useful theoretical tool, because if two dislocations have the same equivalent force, they also emit the same radiation.

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