THE PROPAGATION OF THE PLANETARY - SCALE WAVE DISTURBANCES IN THE IONOSPHERE

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Abstract

Using the analogy method the frequencies of new modes of electromagnetic planetary-scale waves (with wavelength 10^3 km and more) having weather forming nature are found at different ionospheric altitudes. This method gives a possibility to determine a spectra of ionospheric electromagnetic perturbations directly from the dynamic equations without solving the general dispersion equation. It is shown, that the permanently acting factor - latitude variation of the geomagnetic field - generates fast and slow weakly damping planetary electromagnetic waves in both E and F layers of the ionosphere. The waves propagate eastward and westward along the parallels. The fast waves have phase velocities $(1-5)km/s$ and frequencies $10^{-1}-10^{-4}s^{-1}$. The slow waves propagate with the velocities of local winds and have frequencies $10^{-4}-10^{-6}s^{-1}$. The waves generate geomagnetic pulsations of magnitude of order of hundred nanoTesla. The properties and parameters of the theoretically studied electromagnetic waves are in agreement with those of large-scale ultra-low frequency perturbations observed experimentally in the ionosphere.

Key words and phrases: Ionospheric plasma; planetary waves; inhomogeneous geomagnetic field.On the new modes of planetary-scale electromagnetic waves in the ionosphere.

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1. *Introduction*

Numerous ground-based and satellite observations show that the background global planetary-scale electromagnetic wavy perturbations ($\geq 10^3km$) regularly exist in the ionosphere at any season of the year. The observations verify [5,17] the presence of slow (with phase velocities equal to local winds velocities), long-period (a few days and more) and large-scale waves (with wavelength $\lambda \sim 10^3-10^4km$) in the E-layer of the ionosphere. At difference from usual weather forming planetary Rossby waves, these waves cause substantial disturbances of the geomagnetic field (up to ten nanotesla (nT)). Ionospheric observations at the middle latitudes of E-layer verify the existence of fast large-scale electromagnetic perturbations too [3,16]. They propagate along the latitude circles of the Earth with velocities from a few hundreds m/s to a few tens km/s. Their periods vary in the range from a few minutes to a few hours, wavelength is of order of $10^3 km$ and more, amplitude is tens hundreds nT. The phase velocities of these perturbations differ by magnitude at daily and nightly conditions in E-layer of the ionosphere.

These waves have mainly zonal character and are revealed especially during magnetic storms and sub-storms [10], earthquakes [11], artificial explosions [2] and so on. They play an important role in the large-scale synoptic processes and give the possibility to get the valuable information about external sources and dynamical processes taking place in the ionosphere during this period.

Thus, the main problem is to find the factors, generating the background planetary-scale electromagnetic waves in different layers of the ionosphere. It will be shown bellow, that the weather forming planetary electromagnetic waves exist due to the latitude inhomogeneity of the geomagnetic field in the ionosphere.

2. *Formulation of the problem and basic equations*

Ionosphere represents a partially ionized triple component plasma. To describe it we use quasi-hydrodynamic equations, which differ from hydrodynamic equations by the presence of friction force, caused by the collision of different particles [1,4,20]. Quasi-hydrodynamic equations describe the flows, electromagnetic currents and all diffusive processes in the ionospheric plasma. However, the diffusive processes, compressibility and inhomogeneity of the atmosphere play a secondary role for considered large-scale ionospheric perturbations (wavelength $\lambda \geq 10^3 km$). Thus, we can substantially simplify these equations and obtain the following system of equations [4,7,14,20]:

$$\rho_n \frac{\partial \overrightarrow{V_n}}{\partial t} = \overrightarrow{F_n} - \rho_n \nu_{in}(\overrightarrow{V_n} - \overrightarrow{V_i}) - \rho_e \nu_{en}(\overrightarrow{V_n} - \overrightarrow{V_e}), \tag{2.1}$$

$$\rho_e \frac{\partial \overrightarrow{V_e}}{\partial t} = \overrightarrow{F_e} - \rho_e \nu_{en}(\overrightarrow{V_e} - \overrightarrow{V_n}) - \rho_e \nu_{ei}(\overrightarrow{V_e} - \overrightarrow{V_i}) - eN\overrightarrow{E} - \frac{eN}{c}\overrightarrow{Ve} \times \overrightarrow{H_0}, \tag{2.2}$$

$$\rho_i \frac{\partial \overrightarrow{V_i}}{\partial t} = \overrightarrow{F_i} - \rho_i \nu_{in}(\overrightarrow{V_i} - \overrightarrow{V_n}) - \rho_e \nu_{ei}(\overrightarrow{V_i} - \overrightarrow{V_e}) + eN\overrightarrow{E} + \frac{eN}{c}\overrightarrow{V_i} \times \overrightarrow{H_0}, \quad (2.3)$$

$$\nabla \overrightarrow{V_n} = 0, \quad \nabla \overrightarrow{V_e} = 0, \quad \nabla \overrightarrow{V_i} = 0. \quad (2.4)$$

Here, indices n, e and i denote molecules (neutral particles), electrons and ions; $\overrightarrow{V}$ is a velocity; $\rho_n = N_n M$, $\rho_e = Nm$, $\rho_i = NM$ are densities; m and M are masses of electrons and ions (molecule), respectively; N_n and N denote concentrations of the neutral particles and plasma; c is light speed; ν_{ei}, ν_{en}, ν_{in} - denote frequencies of collision of electron with ions and molecules and of ions with molecules, respectively; $\overrightarrow{E}$ is a strength of the induced electric field; $\overrightarrow{H_0}$ is a strength of the geomagnetic field; $\overrightarrow{F_n}$,$\overrightarrow{F_e}$,$\overrightarrow{F_i}$ denote nonelectromagnetic forces, in general case containing gradients of pulse flow density tensor; $\nabla\ (\partial/\partial x, \partial/\partial y, \partial/\partial z)$is a nabla operator.

Equations (2.1)-(2.4), state and heat equations and Maxwell equations form close system of equations for each component. For simplification of these equations we take into account the results of experimental observations of the dynamical processes.

In the ionosphere at the heights of $80 - 500km$ ($\eta = N/N_n\tilde{\ }10^{-9} - 10^{-4} << 1$) non-electromagnetic forces $\overrightarrow{F_n}$,$\overrightarrow{F_e}$,$\overrightarrow{F_i}$ are proportional to the densities of medium components and, hence, $\eta << 1$, $\left|\overrightarrow{F_i}\right| \leq \left|\overrightarrow{F_e}\right| << \left|\overrightarrow{F_n}\right|$. So, $\overrightarrow{F_e}$ and $\overrightarrow{F_i}$ cannot induce big currents. The inertia of electrons and ions can be neglected comparing with the inertia of the neutral particles. Taking into account all these circumstances in Eqs. (2.1)-(2.4), we obtain equation of motion of ionospheric medium:

$$\rho_n \frac{\partial \overrightarrow{V_n}}{\partial t} = \overrightarrow{F_n} + \frac{1}{c}\overrightarrow{J} \times \overrightarrow{H_0} \quad (2.5)$$

where $\overrightarrow{J} = eN(\overrightarrow{V_i} - \overrightarrow{V_e})$ is the density of current. Eqs. (2.2) and (2.3) may be rewritten as

$$-\frac{\nu_{en}}{\omega_e}(\overrightarrow{V_e} - \overrightarrow{V_n}) - \frac{\nu_{ei}}{\omega_e}(\overrightarrow{V_e} - \overrightarrow{V_i}) + \overrightarrow{V_D} \times \overrightarrow{h_0} = \overrightarrow{V_e} \times \overrightarrow{h_0}, \quad (2.6)$$

$$-\frac{\nu_{in}}{\omega_i}(\overrightarrow{V_i} - \overrightarrow{V_n}) - \frac{\nu_{ei}}{\omega_e}(\overrightarrow{V_i} - \overrightarrow{V_e}) + \overrightarrow{V_i} \times \overrightarrow{h_0} = \overrightarrow{V_D} \times \overrightarrow{h_0}, \quad (2.7)$$

where $\omega_e = eH_0/mc$ and $\omega_i = eH_0/Mc$ denote cyclotron frequencies of electrons and ions, respectively, $\overrightarrow{V_D} = c\overrightarrow{E} \times \overrightarrow{H_0}/H_0^2$ is the electron drift velocity; $\overrightarrow{h_0} = \overrightarrow{H_0}/H_0$ is the unit vector along the strength of the geomagnetic field. Taking into account, that in the ionosphere $\omega_e \approx 10^7 s^{-1}$,

$\omega_i \approx (1.5-3)\cdot 10^2 s^{-1}$, the collision frequency reaches its maximal value $\nu_{ei} \approx 10^4 s^{-1}$, $\nu_{in} \approx 10^4 s^{-1}$, $\nu_{en} \approx 10^5 s^{-1}$ at heights $80-500 km$ in the lower layer of the ionosphere and quickly decreases in proportion to height, we can conclude, that $\nu_{ei}/\omega_e << 1$, $\nu_{en}/\omega_e << 1$ in E and F layers of the ionosphere. It means, that electron component of the ionospheric plasma is always magnetized in this region of the upper atmosphere. Taking into account these inequalities, Eqs. (2.6) and (2.7) can be reduced to the following form:

$$\overrightarrow{V_D} \times \overrightarrow{h_0} = \overrightarrow{V_e} \times \overrightarrow{h_0} \Longrightarrow \overrightarrow{V_e} = \overrightarrow{V_D} \Longrightarrow \overrightarrow{E} = -\frac{1}{c}\overrightarrow{V_e} \times \overrightarrow{H_0}, \tag{2.8}$$

$$\overrightarrow{V_i} = \overrightarrow{V_n} + \overrightarrow{J} \times \overrightarrow{H_0}/(\rho c \nu_i), \qquad \nu_i = N\nu_{in}/N_n. \tag{2.9}$$

Therefore, in E and F layers of the ionosphere electrons move with electron drift velocity ($\overrightarrow{V_e} = \overrightarrow{V_D}$) and the geomagnetic field $\overrightarrow{H_0}$ is always frozen in electron component ($\partial \overrightarrow{h}/\partial t = \nabla \times \overrightarrow{V_e} \times \overrightarrow{H_0}$), $\overrightarrow{h}$ denotes the perturbation of geomagnetic field.

Multiplying Eq. (2.8) on $\overrightarrow{H_0}$, we obtain important equality $\overrightarrow{E}\cdot\overrightarrow{H_0} = 0$, i.e. the internal electric field, generated in E and F layers of the ionosphere, is always perpendicular to the geomagnetic field $\overrightarrow{H_0}$.

Using Maxwell equations, we get a closure of the system of equations (2.5), (2.8), (2.9):

$$\frac{\partial \overrightarrow{h}}{\partial t} = -c\nabla \overrightarrow{E}, \qquad \overrightarrow{J} = \frac{c}{4\pi}\nabla \times \overrightarrow{h}. \tag{2.10}$$

Excluding $\overrightarrow{E}$ and $\overrightarrow{J}$, using Eq. (10) and dropping index n for velocity and density of the neutral particles, we obtain a system of magneto-hydrodynamic equations for E and F layers of the ionosphere:

$$\frac{\partial \overrightarrow{V}}{\partial t} = -\frac{1}{\rho}\nabla P' + \frac{\rho'}{\rho}\overrightarrow{g} + \overrightarrow{V} \times 2\overrightarrow{\omega}_0 + \frac{\overrightarrow{F_A}}{\rho}, \tag{2.11}$$

$$\frac{\partial \overrightarrow{h}}{\partial t} = \nabla \times \overrightarrow{V_e} \times \overrightarrow{H_o} = \nabla \times \overrightarrow{V} \times \overrightarrow{H_0} - \alpha\rho\nabla \times \frac{\overrightarrow{F_A}}{\rho} + \nabla \times \frac{1}{\nu_i}\frac{\overrightarrow{F_A}}{\rho} \times \overrightarrow{H_0}, \tag{2.12}$$

where:

$$\frac{\overrightarrow{F_A}}{\rho} = \frac{1}{\rho c}\overrightarrow{J} \times \overrightarrow{H_0} = \frac{1}{4\pi\rho}\nabla \times \overrightarrow{h} \times \overrightarrow{H_0} \approx \overrightarrow{V} \times 2\overrightarrow{\Omega_i} \times -\overrightarrow{V_D} \times 2\Omega_i = \overrightarrow{u} \times 2\overrightarrow{\Omega_i}; \tag{2.13}$$

$\sigma_H = e^2N\left[\omega_e/m(\omega_e^2+\nu_e^2) - \omega_i/M(\omega_i^2+\nu_{in}^2)\right]$ is the Hall conductivity; Hall parameter α in general case is $\alpha = c^2/\overrightarrow{H_0}\sigma_H$; P' and ρ' are perturbations of gas-kinetic pressure and density of neutral particles, accordingly; $\overrightarrow{g}$ is a gravity acceleration; $\overrightarrow{\omega_0}$ is an angular velocity of the Earth rotation; $\overrightarrow{u} = \overrightarrow{V} - \overrightarrow{V_D}$. In E-region of the ionosphere we have $\omega_e >> \nu_{en}$, $\omega_i << \nu_{in}$ and $\alpha = c/(eN)$ (Hall's conductivity disappear higher than $150km \quad \sigma_H = 0$).

From Eq. (2.13) follows that Ampere electromagnetic force $\overrightarrow{F_A}$, acting on a unite mass of medium $\overrightarrow{F_A}/\rho = \overrightarrow{u} \times 2\overrightarrow{\Omega}_i$, has the same structure as the Coriolis acceleration $\overrightarrow{V} \times 2\overrightarrow{\omega_0}$. Therefore, Amper force must act on atmospheric - ionospheric medium similarly to Coriolis force. Similarity of Amper and Coriolis forces means, that new modes of large-scale electromagnetic oscillations must be generated due to inhomogeneity of the geomagnetic field $\overrightarrow{H}$ as well as Rossby-type usual planetary waves are generated due to inhomogeneity of angular velocity of the Earth rotation $\overrightarrow{\omega_0}$. In this case, as it will be shown bellow, the first term of the electromagnetic force $\overrightarrow{F_A}$, caused by the velocity of the medium motion (dynamo field $\overrightarrow{E}_d = \overrightarrow{V} \times \overrightarrow{H_0}/c$), generates slow Rossby-type electromagnetic wave; second term of electromagnetic force, appearing due to vortex electric field $\overrightarrow{E_V} = -\overrightarrow{V_D} \times \overrightarrow{H_0}/c$, generates fast electromagnetic wave.

Estimations show, that for planetary scale perturbations ($L \sim 10^3 - 10^4 km$, further we shall be interested in these perturbations) in E-region of the ionosphere magnetic Reynolds number $Re_m = \omega L^2/\nu_H \sim 1/\alpha$, where L and ω are characteristic linear scale and frequency of perturbations, $\nu_H = c^2/(4\pi\sigma_H)$ reaches the value ($Re_m \sim 20$) sufficiently small. Therefore, it is necessary to keep Hall term (α) of induction equation, but the last term of Eq. (2.12) can be neglected due to condition $\sigma_H >> \sigma_\perp \approx \sigma_H\omega_i/\nu_{in}$ (where $\sigma_\perp$ is the transversal conductivity). In F-region of the ionosphere, where Hall effect is not important, the last term of Eq. (2.12) also can be neglected for planetary-scale perturbations in the first approximation as far as Reynolds number $Re_\perp = \omega L^2/\nu_\perp$ ($\nu_\perp = (c^2/(4\pi\sigma_\perp))$) is of order of 10^2. Observations show, that the planetary waves propagate over great distances in the ionosphere without substantial changes [3,6,16,17]. For planetary scale waves the latitude variations of angular velocity of the Earth rotation $\overrightarrow{\omega_0}(\theta)$ and the geomagnetic field $\overrightarrow{H_0}$ (where θ and θ' are geographical and geomagnetic latitudes) can not be neglected. Therefore, for such large-scale perturbations we must use Helmholtz equation of velocity vortex, which takes into account latitude effects of vectors $\overrightarrow{\omega}_0$ and $\overrightarrow{H}_0$, instead of equation of motion (2.11), as well as in dynamical meteorology [9,12,14]. Helmholtz equation is obtained from Eq. (2.11) by using on its both sides an operator $curl = \nabla\times$. Compressibility and temperature stratification of the atmosphere, as we mentioned above, play a secondary role for such disturbances [9,14,15].

Hence, for E and F layers of the ionosphere magneto - hydrodynamic Eqs. (2.11)-(2.13) may be written in the following form:

$$\frac{\partial \nabla \times \overrightarrow{V}}{\partial t} = \nabla \times \overrightarrow{V} \times 2\overrightarrow{\omega_0} + \frac{1}{4\pi\rho} \nabla \times \nabla \times \overrightarrow{h} \times \overrightarrow{H_0}, \tag{2.14}$$

$$\frac{\partial \overrightarrow{h}}{\partial t} = \nabla \times \overrightarrow{V} \times \overrightarrow{H_0} - \frac{\alpha}{4\pi} \nabla \times \nabla \times \overrightarrow{h} \times \overrightarrow{H_0}, \tag{2.15}$$

$$\nabla \cdot \overrightarrow{V} = 0, \quad \nabla \overrightarrow{h} = 0 \tag{2.16}$$

Here $\overrightarrow{H_0} = H_{0y}\overrightarrow{e_z} + H_{0z}\overrightarrow{e_z}$, $H_{0y} = -H_p \sin\theta'$, $H_{0z} = -2H_p \cos\theta'$, $H_p = 3.2 \cdot 10^{-5}T$; $2\overrightarrow{\omega_0} = 2\omega_{0y}\overrightarrow{e_y} + 2\omega_{0z}\overrightarrow{e_z}$, $2\omega_{0y} = 2\omega_0 \sin\theta$, $2\omega_{0z} = 2\omega_0 \cos\theta$, $\omega_0 = 7.3 \cdot 10^{-5}s^{-1}$ ($\overrightarrow{H_0}$ always has a north-south direction, $\overrightarrow{\omega_0}$ is directed opposite to $\overrightarrow{H_0}$); $\overrightarrow{e_x}$, $\overrightarrow{e_y}$, $\overrightarrow{e_z}$ denote unite vectors along x, y, z axes, respectively; θ' is geomagnetic colatotude and θ - geographical colatitude.

Close system of Eqs. (2.14) and (2.15) contains six scalar equations and gives a possibility to calculate six unknown quantities: $\overrightarrow{V_x}$, $\overrightarrow{V_y}$, $\overrightarrow{V_z}$, $\overrightarrow{h_x}$, $\overrightarrow{h_y}$, $\overrightarrow{h_z}$. After determining the values of $\overrightarrow{V}$ and $\overrightarrow{h}$, pressure P' will be determined from Eq. (2.11) in quadrature (as far as $\rho' = 0$); current density $\overrightarrow{J}$ and electric field $\overrightarrow{E}$ are calculated from Maxwell Eqs. (2.10); electron velocity is determined from the expression $\overrightarrow{V_e} = \overrightarrow{V_D}$, ion velocity is determined from the formula (2.9). Thus, the initial - boundary problem of large-scale dynamics of triple component plasma for E and F layers of the ionosphere in linear approximation is solved completely.

3. *Large-scale wavy perturbations*

The discussed planetary waves have wavelength of order of the Earth radius. Therefore, it is natural to consider a creation of large-scale perturbations in the Earth atmosphere in spherical coordinate system. However, the mathematical difficulties, raised by theoretical investigation of obtained equations, oblige us to consider the problem in standard coordinate system [8,15,14,15]. In this system x-axis is directed to the east towards the parallels, y-axis - to the north along meridian, z-axis is directed vertically up (local Cartesian coordinate system). Length elements dx, dy, dz are connected with the parameters of the spherical coordinate system λ, θ, r by the following approached formulas: $dx = r_0 \sin\theta d\lambda$, $dy = -r_0 d\theta$, $dz = dr$. Velocities are equal: $V_x = V_\lambda$, $-V_y = V_\theta$, $V_z = V_r$. Here, θ is an adjunction to the geographical latitude (co-latitude), λ is a longitude,r_0 is the Earth radius, r is a distance from the center of the Earth along the Earth radius. This system is not equivalent to the ordinary Cartesian system of reference as far as directions of the axes vary with the atmospheric particle motion from one point to another. However, for large-scale processes in thermo-hydrodynamic equations of atmosphere the terms, connected with spatial variations of coordinate axes, may be dropped in the first approximation [12,15,18]. Therefore, the equation of motion in spherical coordinate system (taking into account connections between coordinates, mentioned above) has the same form as in the Cartesian system of reference. This procedure simplifies the problem and investigation of dynamics of large-scale processes in the atmosphere [12,15,18] and therefore, it will be used also for magnetoactive ionospheric medium.

The method of frozen-in coefficients in dynamic equations will be also used below. This method is known as approximation [8,9,12,15] in spherical hydrodynamics and meteorology. In this approximation the parameters $\omega_0(\theta)$, $\nabla\omega_0(\theta)$, $H_0(\theta')$, $\nabla H_0(\theta')$ are constant at integration of dynamical equations taking into account $\theta = \theta_0$, $\theta'.{=}\theta'_0$. Medium motion is considered near θ_0 and θ'_0, i.e. average values of adjunction of geographical φ_0 and geomagnetic φ'_0 latitudes, respectively. In this case, dynamical equations transform into equations with constant coefficients, which may be investigated by plane wave method. Application of β- approximation (or β-plane) leads to simple results, which gives a possibility to reveal more important features of motion on a rotating sphere, which differs from the motion on a rotating plane. Further we guess, that geographical latitude φ coincides with the geomagnetic latitude φ' i.e. $\theta = \theta'$, $\theta_0 = \theta'_0$.

Now we introduce vectorial potential $\overrightarrow{h}/(\alpha\rho) = \nabla \times \overrightarrow{U}$, then we find:

$$\nabla \times \frac{\overrightarrow{h}}{\alpha\rho} = \nabla \times \nabla \times \overrightarrow{U} = \nabla\left(\nabla \cdot \overrightarrow{U}\right) - \triangle \overrightarrow{U}. \tag{3.1}$$

Without loss of generality, we can assume, that $\nabla \cdot \overrightarrow{U} = 0$. This is Lorenz calibration condition, which guarantees a uniqueness of the solution for the vectorial potential $\overrightarrow{U}$. Seeking the solution of Eqs. (2.14) and (2.15) as a plane wave $V, h \sim \exp\left\{i(\overrightarrow{k}\,\overrightarrow{r} - \omega t)\right\}$, where $\overrightarrow{k}$ is the wave vector, ω is the frequency of perturbation, from Eq. (3.1) we obtain $\nabla \times \overrightarrow{h}/(\alpha\rho) = k^2\overrightarrow{U}$. Taking it into account, Eqs. (2.14) and (2.15) may be written in the form:

$$\frac{\partial}{\partial t}\nabla \times \overrightarrow{V} = \nabla \times \overrightarrow{V} \times 2\overrightarrow{\omega_0} - \nabla \times \overrightarrow{U} \times 2\overrightarrow{\Omega_H}, \tag{3.2}$$

$$\frac{\partial}{\partial t}\nabla \times \overrightarrow{U} = \nabla \times \overrightarrow{V} \times 2\overrightarrow{\Omega_0} + \nabla \times \overrightarrow{U} \times 2\overrightarrow{\Omega_H}, \tag{3.3}$$

where $2\overrightarrow{\Omega_H} = -\frac{ck^2}{4\pi eN}\overrightarrow{H_0}$, $2\overrightarrow{\Omega_0} = \frac{Ne}{N_n Mc}\overrightarrow{H_0}$.

Eqs. (3.1) and (3.2) show, that electromagnetic waves must be generated by hydrodynamic and electromagnetic interaction on triple-component ionospheric plasma. From these equations follow, that changing of velocity vortex $\nabla\times\overrightarrow{V}$ and vectorial potential vortex $\nabla\times\overrightarrow{U}$ occurs under the action of Coriolis $F_c = \rho\overrightarrow{V}\times 2\overrightarrow{\omega_0}$ and electromagnetic gyroscopic $\overrightarrow{F_H} = \rho\overrightarrow{V}\times 2\overrightarrow{\Omega_H}$, $\overrightarrow{F_0} = \rho\overrightarrow{V}\times 2\overrightarrow{\Omega_0}$ forces. Solenoidal character automatically is taken into account by $\overrightarrow{U}$ and $\overrightarrow{V}$ vectors. Eqs. (18) and (19) represent close system of equations, describing interaction of two incompressible fluids, moving with velocities $\overrightarrow{V}$ and $\overrightarrow{U}$ under the action of three gyroscopic forces mentioned above. In general case Eqs. (3.2) and (3.3) are of the sixth order with respect to time and corresponding dispersion equation have four nonzero roots for frequency. Two zero frequencies ($\partial/\partial t \sim \omega = 0$) correspond to hydrodynamic and electromagnetic equilibrium in the unperturbed state.

As far as $\overrightarrow{U}$ has velocity dimension m/s, $\overrightarrow{\Omega_H}$ and $\overrightarrow{\Omega_0}$ have dimensions s^{-1}, differential Eq. (3.2) coincides with Eq. (3.3) replacing $\overrightarrow{V}$ by $\overrightarrow{U}$ and $\overrightarrow{\omega_0}$ by $\overrightarrow{\Omega_0} + 2\overrightarrow{\Omega_H}$. Coincidence of these differential equations means that they must describe similar physical phenomena. It must be mentioned, that the analogy method reveals fundamental discoveries in quantum mechanics and in different areas of theoretical physics.

It will be shown, that application of analogy method in general case gives a possibility to seek an electromagnetic analog of atmospheric waves in E and F layers of the ionosphere without solving Eqs. (3.2) and (3.3). For illustration we consider a few particular cases for the system of Eqs. (3.2) and (3.3).

1) For E-region of the ionosphere, $\Omega_0 << \omega_0$, in the right side of Eq. (3.2) first term exceeds second one, vice versa in Eq. (3.3). In this case formulas (3.2) and (3.3) give a close system of equations for $\overrightarrow{V}$ and $\overrightarrow{U}$:

$$\frac{\partial}{\partial t}\nabla\times\overrightarrow{V} = \nabla\times\overrightarrow{V}\times 2\overrightarrow{\omega_0}, \tag{3.4}$$

$$\frac{\partial}{\partial t}\nabla\times\overrightarrow{U} = \nabla\times\overrightarrow{U}\times 2\overrightarrow{\Omega_H} \tag{3.5}$$

Eqs. (3.4) and (3.5) show, that $2\overrightarrow{\omega_0}$ and $2\overrightarrow{\Omega_H}$ are frozen in the field of $\overrightarrow{V}$ and $\overrightarrow{U}$ vectors.

It is well known in dynamical meteorology, that for small-scale ($L <<$ $10^3 km$) processes Eq. (3.4) has the general solution - three-dimensional inertial waves, satisfying the dispersion equation [8,12,15]:

$$\omega = \omega_I = \frac{(2\overrightarrow{\omega_0}\cdot\overrightarrow{k})}{k}, \tag{3.6}$$

where $k = \sqrt{k_x^2 + k_y^2 + k_z^2}$. For large scale ($L \sim 10^3 - 10^4 km$) processes equation (3.4) has an exact solution - slow planetary Rossby waves, satisfying dispersion equation [9,14,15]:

$$\omega = \omega_R = -\beta \frac{k_x}{k_x^2 + k^{y2}}, \tag{3.7}$$

where: $\beta = \partial 2\omega_{0z}/\partial y = 2\omega_0 \sin\theta_0/R$ is the Rossby parameter, $\partial/\partial y = -r_0^{-1}\partial/\partial\theta$. Eq. (3.4) does not contain any new information about atmospheric waves. It is a cubic equation with respect to time and has nonzero proper frequencies ω_I and ω_R. The third root - zero frequency, as it is mentioned above, corresponds to quasi-static and quasi-geostrophic equilibrium ($\overrightarrow{V_g} = 2\overrightarrow{\varpi_0} \times \nabla\overrightarrow{P_0}/(4\omega_0^2\rho)$ is geostrophic wind velocity) state of the atmosphere; $\overrightarrow{P_0}$ is the equilibrium pressure.

Using the analogy method, without solving Eq. (3.5) and utilizing only expression (3.6), we can conclude, that in electromagnetic approximation the analogy of the small-scale inertial waves in the ionosphere is the well known atmospheric whistle (helicons) modes:

$$\omega = \omega_h = -\frac{(2\overrightarrow{\Omega_H} \cdot \overrightarrow{k})}{k} = \frac{ck(\overrightarrow{k} \cdot \overrightarrow{H_0})}{4\pi eN} \tag{3.8}$$

Sign "$-$" denotes opposite directions of $\overrightarrow{\varpi_0}$ and $\overrightarrow{H_0}$ vectors .

For large-scale processes ($L \sim 10^3 \sim 10^4 km$), when latitude variation of the geomagnetic field $\overrightarrow{H_0}$ is negligible, electromagnetic analogy of Rossby waves (3.8) must exist in E-region of the ionosphere

$$\omega = \omega_H = -\beta_H \frac{k_x}{k_x^2 + k_y^2} = \frac{c\beta_1}{4\pi eN} k_x, \tag{3.9}$$

where $\beta_H = \partial 2\Omega_{Hz}/\partial y = -(ck^2/4\pi eN)\cdot\partial H_{0z}/\partial y = -c(k_x^2+k_y^2)/(4\pi eNv\beta_1)$; $\beta_1 = \partial H_{0z}/\partial y - 2H_p \sin\theta_0/r_0$. Taking into account the both components of the geomagnetic field, we get:

$$\omega_H = \frac{\alpha}{4\pi}\sqrt{\beta_1^2 + \beta_2^2}k_x = \frac{cH_p}{4\pi eN}\frac{\sqrt{1 + 3\sin^2\theta}}{r_0}k_x, \tag{3.10}$$

where $\beta_2 = \partial H_{0y}/\partial y$. This is a new mode of proper oscillations in E-region of the ionosphere.

Numerical calculations of planetary wave parameters were carried out using (3.10) models of the ionosphere and the neutral atmosphere [19] for low and high sun activity. Numerical calculations show, that at $\theta = 45^0$, at a heights of $90 - 150$ km, phase velocity of waves $C_H = \omega_H/k_x$ vary from 4 to $1,4$ km/s at night, and from 400 to 800 m/s in the daytime. Periods $T_H = \lambda/C_H$ at $\lambda = 2 \cdot 10^3 km$ vary in the interval of $(1,5 - 6)$

hours in the daytime and $(4-12)$ minutes at night. Perturbation of the geomagnetic field of these waves $h_H = H_p\sqrt{1+3\sin^2\theta}\xi_e/r_0$ (where ξ_e is electron displacement) is 8 and 80 nT at $\xi_e = 0,1km$ and $\xi_e = 1km$. The influence of exospheric temperature on C_H and T_H is insignificant, but is important for magnetic field perturbations. C_H and T_H values substantially differ in the daytime and at night as far as electron concentration in E-region of the ionosphere varies with an order by magnitude during a day.

These oscillations were observed experimentally [3,16] at middle latitudes of E-region of the ionosphere and were extracted as middle-latitude long-period oscillations. Eq. (3.10) does not pose any restrictions for the existence of these perturbations at both high and low latitudes. They are observed especially by the world-wide network of the ionospheric and magnetospheric observatories during the earthquakes, the magnetic storms and artificial explosions [2,10,11].

Eq. (3.5) does not contain any additional information. Now we demonstrate this in general case for high and moderate latitudes ($\overrightarrow{H_0} \approx H_{0z}\overrightarrow{e_z}$). Let us write the equation (3.5) in the following form:

$$\omega \overrightarrow{k} \times \overrightarrow{U} = i2\Omega_{Hz}k_z\overrightarrow{U} - \beta_H U_y \overrightarrow{e_z}, \tag{3.11}$$

$$\overrightarrow{k} \cdot \overrightarrow{U} = 0 \tag{3.12}$$

Multiplying Eq. (3.11) by $\overrightarrow{k}$ vectorially and utilizing Eq. (3.12) we obtain:

$$\overrightarrow{U} = -i\frac{2\Omega_{Hz}}{\omega k^2}k_z\overrightarrow{k} \times \overrightarrow{U} + \frac{\beta_H}{\omega k^2}U_y\overrightarrow{k} \times \overrightarrow{e_z} \tag{3.13}$$

Excluding the expression $\overrightarrow{k} \cdot \overrightarrow{U}$ using (3.12) and taking into account $(\overrightarrow{k} \times \overrightarrow{e_z})_x = k_y$, $(\overrightarrow{k} \times \overrightarrow{e_z})_y = -k_x$, $(\overrightarrow{k} \times \overrightarrow{e_z})_z = 0$, from Eq. (3.13) we obtain the system of equations for U_x, U_y and U_z components:

$$\left(1 - \frac{\omega_h^2}{\omega^2}\right)U_x = \frac{\beta_H}{\omega k^2}k_y U_y, \tag{3.14}$$

$$\left(1 - \frac{\omega_h^2}{\omega^2}\right)U_y = -\frac{\beta_H}{\omega k^2}k_x U_y, \tag{3.15}$$

$$\left(1 - \frac{\omega_h^2}{\omega^2}\right)U_z = i\frac{2\Omega_{Hz}k_z}{\omega k^2}\beta_H U_y. \tag{3.16}$$

From the expression (3.15) it follows, that

$$\left(1-\frac{\omega_h^2}{\omega^2}+\frac{\beta_H k_x}{\omega k^2}\right)U_y=0, \tag{3.17}$$

where $\omega_h = ckk_zH_{0z}/(4\pi eN)$. So, we have two cases $U_y \neq 0$ or $U_y = 0$.

a) At $U_y \neq 0$, in Eq. (3.17) the expression in the round brackets tends to zero

$$\left(1-\frac{\omega_h^2}{\omega^2}+\frac{\beta_H k_x}{\omega k^2}\right)=0 \quad or \quad \left(1-\frac{\omega_h^2}{\omega^2}\right)=-\frac{\beta_H k_x}{\omega k^2} \tag{3.18}$$

Substituting this expression into (3.14) we obtain:

$$-\frac{\beta_H}{\omega k^2}k_xU_x=\frac{\beta_H}{\omega k^2}k_yU_y.$$

From this it follows, that $k_xU_x + k_yU_y = 0$. Taking into account Eq. (3.12) $k_xU_x + k_yU_y + k_zU_z = 0$, we get $k_zU_z = 0$. Therefore, if $k_z = 0$, from formula (3.16) we obtain $U_z = 0$. Thus, k_z, U_z and ω_h vanish simultaneously at $U_y \neq 0$. If U_x and U_y are nonzero, then the dispersion equation (3.18) gives only an analogy of Rossby wave

$$\omega=\omega_H=-\beta_H\frac{k_x}{k_x^2+k_y^2}=\frac{c\beta_1}{4\pi eN}k_x.$$

b) Left-hand sides of Eqs. (3.14) and (3.16) tend to zero at $U_y = 0$.

$$\left(1-\frac{\omega_h^2}{\omega^2}\right)U_x=0; \quad \left(1-\frac{\omega_h^2}{\omega^2}\right)U_z=0.$$

If $U_x = 0$, $U_z = 0$, we have trivial zero solution, which corresponds to equilibrium state, when electric drift velocity and geostrophic wind velocity are equal [7]. At $U_x \neq 0$ and $U_z \neq 0$, we obtain dispersion equation for helicons $\omega = \omega_h = ckk_zH_{0z}/(4\pi eN)$.

Analysis shows, that electromagnetic planetary $C_H = \omega_H/k_x$ wave is a unique solution of Eq. (3.5) at $U_y \neq 0$ and helicons automatically are excluded. Helicon waves are unique solution at $U_y = 0$, C_H waves are filtered out. Using the analogy method this proof may be carried out directly for inertial and planetary Rossby waves, i.e. for Fridman equation (3.4).

2) In F-region of the ionosphere, $\Omega_0 >> \omega_0$, we can neglect the first term in comparison with the second one in the right-hand side of Eq. (3.2) and vice versa in Eq. (3.3):

$$\frac{\partial}{\partial t}\nabla\times\overrightarrow{V}=-\nabla\times\overrightarrow{U}\times 2\overrightarrow{\Omega_H}, \tag{3.19}$$

$$\frac{\partial}{\partial t}\nabla\times\overrightarrow{U}=\nabla\times\overrightarrow{V}\times 2\overrightarrow{\Omega_0} \tag{3.20}$$

It is easy to show, that the system of Eqs. (3.19) and (3.20) does not contain Hall parameter and therefore, this system can be applied for investigation of electromagnetic processes in F-region of the ionosphere.

For small-scale processes, when latitude variations of the geomagnetic field $\overrightarrow{H_0}$ is negligible, parameters $2\Omega_0$ and $2\Omega_H$ become constant and the system of Eqs. (35) and (36) can be solved in general case. Indeed, using transversal condition of the waves $(\overrightarrow{k}\cdot\overrightarrow{U})=0$, $(\overrightarrow{k}\cdot\overrightarrow{V})=0$ from Eqs. (3.19) and (3.20) we get:

$$\omega\overrightarrow{k}\times\overrightarrow{V}=-i(\overrightarrow{k}\cdot 2\overrightarrow{\Omega_H})\overrightarrow{U};\qquad \omega\overrightarrow{k}\times\overrightarrow{U}=i(\overrightarrow{k}\cdot 2\overrightarrow{\Omega_0})\overrightarrow{V}. \tag{3.21}$$

In F-region of the ionosphere eliminating $\overrightarrow{U}$ and $\overrightarrow{V}$, we obtain dispersion equation for modified Alfven waves:

$$\omega^2=-\frac{(\overrightarrow{k}\cdot 2\overrightarrow{\Omega_0})}{k}\frac{(\overrightarrow{k}\cdot 2\overrightarrow{\Omega_H})}{k},$$

from which it follows, that

$$\omega_{1,2}=\pm\sqrt{\eta}\frac{(\overrightarrow{k}\cdot\overrightarrow{H_0})}{\sqrt{4\pi MN}}=\pm\sqrt{\eta}\omega_A. \tag{3.22}$$

Non-dimensional parameter $\eta=N/N_n$ denotes a degree of ionization of plasma , $\omega_A=(\overrightarrow{k}\cdot\overrightarrow{H_0})/\sqrt{4\pi MN}$ is Alfven frequency. Modified Alfven waves are slow waves as far as parameter varies in the interval of $(10^{-7}-10^{-3})$ for F-region of the ionosphere $(200-500)km$. Dispersion equation (3.22) has two roots for positive and negative propagation directions. Group velocity of these perturbations is directed along the force lines of the geomagnetic field $\overrightarrow{H_0}$.

Similarly to (3.22), from Eqs. (3.19) and (3.20) we obtain only one root of dispersion equation, describing propagation of zonal perturbations along latitude circles (along x-axis, directed along parallel) for large-scale processes, when latitude variations of the geomagnetic field $\overrightarrow{H_0}$ are not negligible:

$$\omega=\omega_n=\sqrt{\eta}\frac{H_p}{\sqrt{4\pi MN}}\frac{\sqrt{1+3\sin^2\theta}}{r_0} \tag{3.23}$$

Calculations show, that the phase velocity of waves $C_n=\omega_n/k_x$ are in the range of $(20-1400)km/s$ at the heights of $200-500km$, $\lambda=2\cdot 10^3 km$, $\theta=45^0$, exosphere temperature is $T_{exos}=600^0K$; and in the range of

$(10-50)km/s$ at $T_{exos} = 2600^0K$. Period of these waves $T_n = 2\pi/\omega_n$ does not depend on wavelength and varies in the interval of $(105-3)s$ at $T_{exos} = 600^0K$ and $(210-40)s$ at $T_{exos} = 2600^0K$. Magnetic pulsations, induced by these waves have the same order of magnitude as C_H waves, $h_n = h_H$. Existence of Coriolis force and the ordinary Rossby waves in F-region of the ionosphere leads to dispersion relation $(\omega/k_x)^2 = C_n^2(1-\omega_R/\omega)^{-1}$. Periods, phase velocities and amplitudes of geomagnetic pulsations for C_n waves in the middle-latitude ionosphere are in agreement with observation data of both middle-latitude and large-scale electromagnetic perturbations, generated in F-region of the ionosphere at powerful earthquakes and magnetic storms [3,10,11].

3) Now we consider frequency band $\omega << 2\Omega_{H0}$. In Eq. (3.3) the left-hand side can be neglected in comparison with free terms (which is fulfilled for potential electric fields):

$$\frac{\partial}{\partial t}\nabla \times \overrightarrow{V} = \nabla \times \overrightarrow{V} \times 2\overrightarrow{\omega_0} - \nabla \times \overrightarrow{U} \times 2\overrightarrow{\Omega_H}, \tag{3.24}$$

$$0 = \nabla \times \overrightarrow{V} \times 2\overrightarrow{\Omega_0} + \nabla \times \overrightarrow{U} \times 2\overrightarrow{\Omega_H} \tag{3.25}$$

Eliminating $\nabla \times \overrightarrow{U} \times 2\overrightarrow{\Omega_H}$, we obtain generalized Fridman equation for vorticity:

$$\frac{\partial}{\partial t}\nabla \times \overrightarrow{V} = \nabla \times \overrightarrow{V} \times (2\overrightarrow{\omega_0} + 2\overrightarrow{\Omega_0}) - \nabla \times \overrightarrow{V} \times 2(\overrightarrow{\omega_0} + \eta\omega_i), \tag{3.26}$$

where ω_i is an ion gyrofrequency.

From Eq. (3.26), as in case 1), it follows, that there must exist two classes of solutions:

1) small-scale modified inertial waves, having frequency

$$\omega = \omega_i' = \frac{(2\overrightarrow{\omega_0'} \cdot \overrightarrow{k})}{k}, \tag{3.27}$$

where $2\omega_0' = 2(\omega_0 + \eta\omega_i)$,

2) large-scale planetary Rossby-type waves, having both hydrodynamic and electromagnetic nature (compare with [19]):

$$\omega = \omega_R' = -\beta' \frac{k_x}{k_x^2 + k_y^2} \tag{3.28}$$

Here $\beta' = \beta + \beta_i$, $\beta_i = \eta\partial\omega_{iz}/\partial y$. Calculations show, that phase velocities of $C_R' = \omega_R'/k_x = -\beta'\lambda^2/(4\pi^2)$ - waves are in the range of $(-2+80)m/s$ in the daytime, $T_{exos} = 600^0K$ at the heights of $(90-150)km$ and

$\lambda = 2 \cdot 10^3 km$. For $\lambda = 2 \cdot 10^4 km$ phase velocities change from $(-41)m/s$ to $(+1,8)m/s$ in the daytime and in the range $(-41 \div 11)m/s$ at night. Velocities change from $-3m/s$ to $+60m/s$ in the daytime and from $-2m/s$ to $-1,3m/s$ at night, $\lambda = 2 \cdot 10^3\ km$, $T_{exos} = 2600^0 K$. In this case sign "-" points to the direction of phase velocity from the east to the west, sign "+" from the west to the east. Calculations show, that $\beta' = (N\omega_{iz}/N_n - \Omega_0) 2\sin\theta/r_0$ tends to zero and $C_R' = 0$ in the daytime at a height of $115km$. Parameter β' tends to zero at a height of $150km$ at the nightly ionosphere. Hence, ordinary slow planetary Rossby waves, moving from the west to the east direction in the daytime, will be revealed in the lower E-region at the heights of $90 - 115km$; the fast planetary waves, having electromagnetic nature and moving from the west to the east direction, will be revealed at and higher than critical altitudes. Hall region completely is occupied by the slow Rossby waves at the nightly ionosphere. Hence, magnetic control of planetary waves in the ionosphere depends on critical altitude, where the condition $\beta' = \beta + \beta_i = 0$ is fulfilled. These altitudes may be revealed experimentally at registration of the planetary waves jointly by both ionospheric and magnetospheric observatories. Calculation shows, that periods $T_R' = 2\pi/\omega_R'$ are in the interval from 14 day to 8 hours at the heights of $90 - 150km$, $T_{exos} = 600^0 K$, $\lambda = 2 \cdot 10^3 km$. T_R' varies from 14 day to 2 hours at $T_{exos} = 2600^0 K$. Perturbation of the geomagnetic field runs up to a few tens nT. Parameters of C_R' waves are in a good agreement with observed parameters of planetary electromagnetic waves in E-region of the ionosphere at moderate latitudes in any season of the year [5,6,17].

4. *Conclusion*

The analogy method yields simple and important physical results. Particularly, the investigation of equations (3.2) and (3.3) show, that four normal modes: the small-scale inertial waves, the atmospheric whistles (helicons), the fast large-scale electromagnetic planetary C_H-waves and the slow Rossby-type waves must exist in E-region of the ionosphere. Modified small-scale slow Alfven waves with ω_+ and ω_- frequencies, the fast large-scale electromagnetic planetary waves $C_n = \omega/k_x$ and ordinary slow planetary Rossby waves must exist in F-region of the ionosphere. Two eigen-frequencies and $\omega = 0$ correspond to hydrodynamic and electromagnetic equilibrium state of the ionospheric medium in a background state, where the geostrophic wind velocity coincides with the electric drift velocity.

Existence of the large-scale fast waves C_H (in E-region), C_n (in F-region) and slow Rossby-type planetary waves C_R' (in both E and F-regions) are caused by inhomogeneity of the geomagnetic field $\overrightarrow{H_0}$. The slow waves are generated by polarized electrostatic dynamo field of polarization $\overrightarrow{E_d} = \overrightarrow{V} \times \overrightarrow{H_0}/c$, the fast waves - by vortical electric field $\overrightarrow{E_V} = \overrightarrow{V_D} \times \overrightarrow{H_0}/c$. The frequencies of these waves vary in the range $\omega \sim 1 - 10^{-6} s^{-1}$and occupy both infrasound and ultralow frequency (ULF) bands. Wavelength $\lambda \sim 10^3 - 10^4 km$, and period of oscillation $T \sim 1s - 14$ days. This waves generate pulsations of the geomagnetic field $1 - 10^2 nT$.

In the ionosphere the dynamics of the slow planetary electromagnetic waves are more or less studied experimentally. Experimental investigation of features of the fast large-scale electromagnetic waves must be realized. Formulae (3.11) and (3.24) show, that the fast electromagnetic large-scale ($L \sim 10^3 - 10^4 km$) atmospheric waves in both E and F - regions of the ionosphere have general-planetary character and occupy latitudes from the pole ($\theta = 0$) to the equator ($\theta = \pi/2$).

Fast electromagnetic atmospheric waves at ionospheric altitudes can be revealed experimentally and registered using their specific features:

1) phase velocity - latitude relation has a wide range (phase velocities of these waves are increased from the pole to the equator; they are doubled at the equator).

2) high variation (by magnitude) of electron concentration N substantially increases the phase velocity of waves $C_H = \omega_H/k_x$ in E-region of the ionosphere at nightly conditions (from a few hundred m/s in the daytime to a few tens km/s at night).

3) application of well-known profiles $N(h)$ allows us to calculate uniquely a height distribution of C_H waves in E - region of the ionosphere and, vice versa, from a height distribution of $C_H(h)$ waves we can get the dependence of concentration $N(h)$ on the altitude.

4) altitude variation of the neutral component concentration $N_n(h)$ leads to strong increase of phase velocity of C_n waves (phase velocity of C_n waves increases from a few km/s to $1000 km/s$ at the heights of $200-500 km$) in F-region of the ionosphere.

5) C_H and C_n waves give response to the earthquake, magnetic storms, artificial explosions and magnetic activity of the sun.

6) electromagnetic and large-scale $(10^3 - 10^4 km)$ character of both C_H and C_n waves allows their registration by the world-wide network of the ionospheric and magnetospheric observatories.

In conclusion we can say, that the planetary waves in the ionosphere, unlike to the troposphere, generate highly temporal - varying "weather". The waves occupy large temporal interval from two days and more (slow planetary ω_R' waves) and from a few hours to a few minutes and less (fast planetary ω_H and ω_n- waves).

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