THEORY OF CONNECTIVITY AND APPORTIONMENT OF REPRESENTATIVE ACTIVITY CHAINS IN THE PROBLEM OF DECISION-MAKING CONCERNING EARTHQUAKE POSSIBILITY

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Abstract

The problem solved in our article is connected with the investigation of the possibility of using the Atkin's connectivity theory for coming to a decision of three principal elements of an earthquake: the place, time and power.

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1. *Introduction*

There are two aspects of the problem: physical and informational. The physical aspect means a creation of detailed and sufficiently adequate models of physical process that generates and accompanies an earthquake. This is a problem of great importance. Today the situation becomes complicated by accumulated enormous information regulated hard and lack of its nature analyses, which is so necessary for development of effective and fast methods of processing the information. This is the second aspect of the problem and its main content. Our approach is determined by the following factors: 1) Our vision of the modern state of the problem of earthquake prognoses; 2) The type of precursors typical for the region and available for measurement in real conditions; 3) Nature of information and its analysis; 4) The suitable selection of information processing methods.

At the initial period of gathering and processing the accessible information it is estimated that this information embodies combined probability-possibility nature. Detailed analysis of this information was carried out and its probability-possibility nature was motivated.

Based on these findings, the so-called fuzzy discrimination analysis [1], along with the supplementary connectivity analysis [2],[3], was chosen as the most effective method for processing primary data.

1^0. Primary data, as in discrimination analysis, is presented in the form of frequency matrix [1]

$$\widehat{F} = \begin{array}{c} \begin{array}{cccc} M_1 & M_2 & ... & M_n \end{array} \\ \left(\begin{array}{cccc} f_{11} & f_{12} & ... & f_{1n} \\ f_{21} & f_{22} & ... & f_{2n} \\ ... & ... & ... & ... \\ f_{m1} & f_{m2} & ... & f_{mn} \end{array} \right) \end{array} \begin{array}{c} \\ A_1 \\ A_2 \\ ... \\ A_m \end{array}, \tag{1.1}$$

where M_k $\left(k = \overline{1,n}\right)$ are the values of earthquake power, A_l $\left(l = \overline{1,m}\right)$ - activities (earthquake precursors and also constant characteristics such as seismic regional data or all exactly forecastable ,,triggering" effects etc.), f_{ij} $\left(i = \overline{1,m}; j = \overline{1,n}\right)$ is a relative frequency of activity A_i when power of earthquake is M_j. Analysis of information we speak about is necessary at first for determination of precursors image (the horizontal entries of frequency matrix). As vertical entries we suggest the following intervals of power:

$$[< 3]\,,\; [3;4]\,,\; [4;5]\,,\; [5;6]\,,\; [6;7]\,,\; [> 7] \tag{1.2}$$

These intervals we consider as the values of fuzzy variable the linguistic description of which is

$$,,earthquake" =,,earthquake([< 3])\ also$$

$$,,earthquake\ of\ moderate\ power"([3-5])also$$

$$,,strong\ earthquake"([> 5])$$

So-called anomalies (precursors) in behaviour of some two-dimensional time (t_1, t_2) functions have been considered as activities (for example, anomalies in behaviour of the electrical field strain, vertical current, pressure gradient and so on). The structure of the first component t_1 is: the component presents itself as separate moments of measurement of the function values during a ten-day period before an earthquake, with an interval of an hour. Component t_2 is years when the earthquake occurred. Such structure corresponds to available data. Naturally, other data determines other structure.

Details of determination of the ,,anomaly image" by fuzzy set-theoretic methods are cited in [4].

Thus for fixed value of t_1 the frequency matrix (1.1) has the form:

Table1

		earthquake weak group earthquake (1)	mod erade earthquake (2)		strong earthquake (3)		
power							
		[<3]	[3-4]	[4-5]	[5-6]	[6-7]	[>7}
anomaly of	$[\Delta\xi_{\min}, \Delta\xi^1]$	f_{11}	f_{12}	f_{13}	f_{14}	f_{15}	f_{16}
electrical	$[\Delta\xi^1, \Delta\xi^2]$	f_{21}	f_{22}	f_{23}	f_{24}	f_{25}	f_{26}
field	$[\Delta\xi^r, \Delta\xi^{\max}]$	f_{r+11}	f_{r+12}	f_{r+13}	f_{r+14}	f_{r+15}	f_{r+16}
anomaly of	$[\Delta\eta_{\min}, \Delta\eta^1]$	f_{r+21}	f_{r+22}	f_{r+23}	f_{r+24}	f_{r+25}	f_{r+26}
pressure	$[\Delta\eta^1, \Delta\eta^2]$	f_{r+31}	f_{r+32}	f_{r+33}	f_{r+34}	f_{r+35}	f_{r+36}
changing	$[\Delta\eta^l, \Delta\eta^{\max}]$	f_{r+l+31}	f_{r+l+32}	f_{r+l+33}	f_{r+l+34}	f_{r+l+35}	f_{r+l+36}
...			...			...	
group of	$A^{(1)}$	f_{k1}	f_{k2}	f_{k3}	f_{k4}	f_{k5}	f_{k6}
constant	$A^{(2)}$	f_{k+11}	f_{k+12}	f_{k+13}	f_{k+14}	f_{k+15}	f_{k+16}
...			...			...	
factors	$A^{(s)}$	f_{k+s1}	f_{k+s2}	f_{k+s3}	f_{k+s4}	f_{k+s5}	f_{k+s6}

In this table the precursors (anomalies) are naturally divided into groups: the group of electrical field, pressure group and etc. The whole amplitude of anomaly changing is divided by intervals so that each of them is considered as an activity.

Along with the aforementioned values indicating the strain of electric field, vertical current, pressure, gradient of pressure, magnetic field anomalies, the characteristics of slow motion of the Earth crust, changes in the distribution of resilient wave velocities, data received from observing mineral waters, changes in the concentration of various chemical elements contained in underground waters and very noticeable precursors like the abnormal behaviour of animals (especially fish) are also considered lightning, changes of the level of water in drill holes and „constant" characteristics such as seismological zoning and exactly forecastable "triggering" effects.

2^0. Instead of a frequency matrix $\widehat{F}$ of discrimination analysis consider a „submatrix" corresponding to some interval of power and certain geofield, for example,

$$\widehat{F}^{(k)} = \begin{array}{c} \begin{array}{ccccc} M_{s_1}^{(k)} & M_{s_2}^{(k)} & M_{s_3}^{(k)} & M_{s_4}^{(k)} & M_{s_5}^{(k)} \end{array} \\ \left(\begin{array}{ccccc} f_{s_1t_1} & f_{s_2t_1} & f_{s_3t_1} & f_{s_4t_1} & f_{s_5t_1} \\ f_{s_1t_2} & f_{s_2t_2} & f_{s_3t_2} & f_{s_4t_2} & f_{s_5t_2} \\ f_{s_1t_3} & f_{s_2t_3} & f_{s_3t_3} & f_{s_4t_3} & f_{s_5t_3} \\ f_{s_1t_4} & f_{s_2t_4} & f_{s_3t_4} & f_{s_4t_4} & f_{s_5t_4} \end{array} \right) \end{array} \begin{array}{c} \\ A_{t_1} \\ A_{t_2} \\ A_{t_3} \\ A_{t_4} \end{array}, \quad (k = 1, 2, 3) \qquad (1.3)$$

and replace it by ,,linguistic" variant, where matrix elements are values of linguistic variable. Here we consider a classical (non-fuzzy) case when this variable takes only two values: 1 (the given type of earthquake is accompanied by a certain activity) and 0 (earthquake is not accompanied by this activity).

For example we receive the following matrix:

$$^{+}\widehat{R}_k = \begin{array}{c} \begin{array}{ccccc} M_{s_1}^{(k)} & M_{s_2}^{(k)} & M_{s_3}^{(k)} & M_{s_4}^{(k)} & M_{s_5}^{(k)} \end{array} \\ \left(\begin{array}{ccccc} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \end{array} \right) \end{array} \begin{array}{c} \\ A_{t_1} \\ A_{t_2} \\ A_{t_3} \\ A_{t_4} \end{array}. \qquad (1.4)$$

By Atcin's terminology this matrix is called an incidence matrix; it determines some relation (called ,,connectivity") on the Decarte product $\{M \times \{A\}\}$. Thus vertical entries of this matrix correspond to certain cases of earthquakes and horizontal ones - to relevant activities.

Applying to such incidence matrix the connectivity theory one can pick out representative chains of activities for given earthquake. Consider the procedure in details for the matrix (1.4) where we suppose $k = 2$. Let besides:

$$^{+}\widehat{R}_1 = \begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{pmatrix} \qquad ^{+}\widehat{R}_3 = \begin{pmatrix} 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{pmatrix}. \qquad (1.5)$$

Consideration of these incidence matrices does not remove uncertainty but as we can see it enables to receive the distribution of representative chains by connectivity levels and consequently to design the generalized solution in the form of possibility distribution on the set of possible types of earthquakes. Atkin's method permits to calculate the quantity of connectivity corresponding to any pair of columns or rows of the incidence matrix. This notion of connectivity is derived by viewing the rows or columns of incidence matrix as polyhedra in multi-dimensional space. The connectivity between two polyhedra is given by the number of shared faces. Two points are equivalent to a single face, three points to two faces and so on. Thus earthquakes $M_{s_1}^{(1)}$ and $M_{s_4}^{(1)}$share a single face via activities A_1 and A_3. Activities A_3 and A_4 share two faces via earthquakes $M_{s_2}^{(2)}, M_{s_4}^{(2)}$ and $M_{s_5}^{(2)}$, one face via $M_{u_1}^{(1)}$ and $M_{u_2}^{(1)}$ and have no shared faces in the third group of earthquakes.

The connectivities between earthquakes ${}^{\pm}\widehat{C}_M^{(k)}$, and activities ${}^{\pm}\widehat{C}_A^{(k)}$, are given by

$$ {}^{+}\widehat{C}_M^{(k)} = {}^{+}\widehat{R}_k^{T\,+}\widehat{R}_k - \widehat{\Omega}_M, \; {}^{+}\widehat{C}_A^{(k)} = {}^{+}\widehat{R}_k^{+}\widehat{R}_k^{T} - \widehat{\Omega}_A, \tag{1.6} $$

$$ {}^{-}\widehat{C}_M^{(k)} = {}^{-}\widehat{R}_k^{T\,-}\widehat{R}_k - \widehat{\Omega}_M, \; {}^{-}\widehat{C}_A^{(k)} = {}^{-}\widehat{R}_k^{-}\widehat{R}_k^{T} - \widehat{\Omega}_A, \tag{1.7} $$

where ${}^{+}\widehat{R}_k^{T}$ is the transposed of matrix $\widehat{R}_k$; $\widehat{\Omega}_M$ and $\widehat{\Omega}_A$ are simply matrices with all elements unity and their dimensionality coincides correspondingly with dimensions $\widehat{R}_k^{T}\widehat{R}_k$ and $\widehat{R}_k\widehat{R}_k^{T}$. Thus

$$ {}^{+}\widehat{C}_M^{(k)} = \begin{pmatrix} 3 & 3 & 1 & 2 \\ 3 & 3 & 1 & 2 \\ 1 & 1 & 2 & 1 \\ 2 & 2 & 1 & 3 \end{pmatrix} - \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix} $$

$$ = \begin{array}{cc} \begin{array}{cccc} M_{u_1}^{(1)} & M_{u_2}^{(1)} & M_{u_3}^{(1)} & M_{u_4}^{(1)} \end{array} & \\ \begin{pmatrix} 2 & 2 & 0 & 1 \\ 2 & 2 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 2 \end{pmatrix} & \begin{array}{c} M_{u_1}^{(1)} \\ M_{u_2}^{(1)} \\ M_{u_3}^{(1)} \\ M_{u_4}^{(1)} \end{array} \end{array} \tag{1.8} $$

$$ {}^{+}\widehat{C}_A^{(1)} = \begin{pmatrix} 3 & 1 & 2 & 3 \\ 1 & 2 & 1 & 1 \\ 2 & 1 & 3 & 2 \\ 3 & 1 & 2 & 3 \end{pmatrix} - \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix} \tag{1.9} $$

$$ = \begin{array}{cc} \begin{array}{cccc} A_1 & A_2 & A_3 & A_4 \end{array} & \\ \begin{pmatrix} 2 & 0 & 1 & 2 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 2 & 1 \\ 2 & 0 & 1 & 2 \end{pmatrix} & \begin{array}{c} A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \end{array} \tag{1.10} $$

Analogously,

$$
{}^{+}\widehat{C}_{M}^{(2)} = \begin{pmatrix} 2 & 1 & 0 & 1 & 0 \\ 1 & 2 & 1 & 2 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 2 & 1 & 2 & 1 \\ 0 & 1 & 0 & 1 & 1 \end{pmatrix}, {}^{+}\widehat{C}_{M}^{(3)} = \begin{pmatrix} 3 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & -1 & -1 \\ 2 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 \\ 1 & -1 & 0 & -1 & 1 \end{pmatrix} \tag{1.11}
$$

$$
{}^{+}\widehat{C}_{A}^{(2)} = \begin{pmatrix} 2 & 0 & 1 & 2 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 2 & 1 \\ 2 & 0 & 1 & 2 \end{pmatrix}, {}^{+}\widehat{C}_{A}^{(3)} = \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 3 & 2 & 2 \\ 0 & 2 & 3 & 2 \\ -1 & 2 & 2 & 3 \end{pmatrix} \tag{1.12}
$$

-1 above indicates that earthquakes $M_{v_2}^{(3)}$ and $M_{v_5}^{(3)}$, $M_{v_4}^{(3)}$ and $M_{v_5}^{(3)}$ are totally disconnected. A key feature of our classification of earthquakes and activities is that there is considered evidence for each hypothesis as well as against it. This was the motivation for the positive and negative aspects of connectivity. Essentially positive connectivity examines the connectedness of the unit elements in the incidence matrix, whilst negative connectivity examines zeros. Formulas (1.8)-(1.12) are for positive connectivity. Adduce corresponding expressions for negative connectivity:

$$
{}^{-}\widehat{C}_{M}^{(1)} = \begin{pmatrix} 0 & 0 & -1 & -1 \\ 0 & 0 & -1 & -1 \\ -1 & -1 & 1 & -1 \\ -1 & -1 & -1 & 0 \end{pmatrix}, {}^{-}\widehat{C}_{A}^{(1)} = \begin{pmatrix} 0 & -1 & -1 & 0 \\ 0 & 1 & -1 & -1 \\ -1 & -1 & 0 & -1 \\ 0 & -1 & -1 & 0 \end{pmatrix} \tag{1.13}
$$

$$
{}^{-}\widehat{C}_{M}^{(2)} = \begin{pmatrix} 0 & -1 & -1 & -1 & -1 \\ -1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 \end{pmatrix}, {}^{-}\widehat{C}_{A}^{(2)} = \begin{pmatrix} 3 & 0 & 0 & -1 \\ 0 & 0 & -1 & -1 \\ 0 & -1 & 0 & -1 \\ -1 & -1 & -1 & 0 \end{pmatrix} \tag{1.14}
$$

$$
{}^{-}\widehat{C}_{M}^{(3)} = \begin{pmatrix} -1 & -1 & -1 & -1 & -1 \\ -1 & 1 & 0 & 1 & -1 \\ -1 & 0 & 0 & 0 & -1 \\ -1 & 1 & 0 & 2 & 0 \\ -1 & -1 & -1 & 0 & 1 \end{pmatrix}, {}^{-}\widehat{C}_{A}^{(3)} = \begin{pmatrix} 1 & 0 & 0 & -1 \\ -1 & 1 & -1 & 0 \\ 1 & 0 & 2 & -1 \\ -1 & 0 & -1 & 0 \end{pmatrix} \tag{1.15}
$$

Atkin's theory permits to establish the connectivity not only between two polyhedra but also chains of polyhedra connected with at least some given number of faces, i.e. chains of certain earthquakes or activities. This chains of connection can be determined directly from matrices $^{\pm}\widehat{C}_M^{(k)}$ and $^{\pm}\widehat{C}_A^{(k)}$. Results for our example are cited in tables:

Table2

positive-connectivity		negative-connectivity	
$q-$ connectivity	representative $+\widehat{C}_M^{(1)}-$ chains	$q-$ connectivity	representative $-C_M^{(1)}-$ chains
$q=2$	$\{M_{u_1}^{(1)}, M_{u_2}^{(1)}\}\{M_{u_4}^{(1)}\}$	$q=2$	
$q=1$	$\{M_{u_1}^{(1)}, M_{u_2}^{(1)}, M_{u_4}^{(1)}\}, \{M_{u_3}^{(1)}\}$	$q=1$	$\{M_{u_3}^{(1)}\}$
$q=0$	$\{M_{u_1}^{(1)}, M_{u_2}^{(1)}, M_{u_3}^{(1)}, M_{u_4}^{(1)}\}$	$q=0$	$\{M_{u_1}^{(1)}, M_{u_2}^{(1)}\}\{M_{u_4}^{(1)}\}$

positive-connectivity		negative-connectivity	
$q-$ connectivity	representative $\widehat{C}_A^{(1)}-$ chains	$q-$ connectivity	representative $-C_A^{(1)}-$ chains
$q=2$	$\{A_1, A_4\}, \{A_3\}$	$q=2$	
$q=1$	$\{A_1, A_3, A_4\}, \{A_2\}$	$q=1$	$\{A_2\}$
$q=0$	$\{A_1, A_2, A_3, A_4\}$	$q=0$	$\{A_1, A_3, A_4\}, \{A_2\}$

Table3

positive-connectivity		negative-connectivity	
$q-$ connectivity	representative $+\widehat{C}_M^{(2)}-$ chains	$q-$ connectivity	representative $-C_M^{(1)}-$ chains
$q=2$	$\{M_{u_1}^{(2)}\}\{M_{u_2}^{(2)}, M_{u_4}^{(2)}\}$	$q=2$	
$q=1$	$\{M_{u_1}^{(2)}, M_{u_2}^{(2)}, M_{u_3}^{(2)}, M_{u_4}^{(2)}, M_{u_5}^{(2)}\}$	$q=1$	$\{M_{u_3}^{(2)}\}, \{M_{u_5}^{(2)}\}$
$q=0$	$\{M_{u_1}^{(2)}, M_{u_2}^{(2)}, M_{u_3}^{(2)}, M_{u_4}^{(2)}, M_{u_5}^{(2)}\}$	$q=0$	$\{M_{u_1}^{(2)}, M_{u_2}^{(2)}, M_{u_3}^{(2)}, M_{u_4}^{(2)}, M_{u_5}^{(2)}\}$

positive-connectivity		negative-connectivity	
$q-$ connectivity	representative $+C_A^{(2)}-$ chains	$q-$ connectivity	representative $-\widehat{C}_A^{(2)}-$ chains
$q=3$	$\{A_3\}$	$q=3$	$\{A_1\}$
$q=2$	$\{A_1, A_4\}, \{A_3\}$	$q=2$	$\{A_1\}$
$q=1$	$\{A_1, A_3, A_4\}$	$q=1$	$\{A_1\}$
$q=0$	$\{A_1, A_2, A_3, A_4\}$	$q=0$	$\{A_1, A_2, A_3\}, \{A_4\}$

Table4

positive-connectivity		negative-connectivity	
$q-$ connectivity	representative $+\widehat{C}_M^{(3)}-$ chains	$q-$ connectivity	representative $-C_M^{(3)}-$ chains
$q=3$	$\{M_{v_1}^{(3)}\}$	$q=3$	
$q=2$	$\{M_{v_1}^{(3)}, M_{v_3}^{(3)}\}$	$q=2$	$\{M_{v_4}^{(3)}\}$
$q=1$	$\{M_{v_1}^{(3)}, M_{v_2}^{(3)}, M_{v_3}^{(3)}, M_{v_5}^{(3)}\}$	$q=1$	$\{M_{v_2}^{(3)}, M_{v_4}^{(3)}\}$
$q=0$	$\{M_{v_1}^{(3)}, M_{v_2}^{(3)}, M_{v_3}^{(3)}, M_{v_4}^{(3)}, M_{v_5}^{(3)}\}$	$q=0$	$\{M_{v_1}^{(3)}\}\{M_{v_2}^{(3)}, M_{v_3}^{(3)}, M_{v_4}^{(3)}, M_{v_5}^{(3)}\}$
positive-connectivity		negative-connectivity	
$q-$ connectivity	representative $+C_A^{(3)}-$ chains	$q-$ connectivity	representative $-\widehat{C}_A^{(3)}-$ chains
$q=3$	$\{A_2\},\{A_3\},\{A_4\}$	$q=3$	
$q=2$	$\{A_2, A_3, A_4\}$	$q=2$	$\{A_3\}$
$q=1$	$\{A_2, A_3, A_4\}$	$q=1$	$\{A_1\},\{A_2\},\{A_3\}$
$q=0$	$\{A_1, A_2, A_3, A_4\}$	$q=0$	$\{A_1, A_2, A_3, A_4\}$

Here q indicates the level of connectivity (number of faces). Brackets enclose the polyhedra connected at that level. Thus earthquake of first group $M_{u4}^{(1)}$ is selfconnected at level 2; at that level $M_{u1}^{(1)}$ and $M_{u2}^{(1)}$ are connected. At level 2 $M_{u4}^{(1)}$ and $\left\{M_{u1}^{(1)}, M_{u2}^{(1)}\right\}$ are isolated and they are joined at level 1.

$\widehat{C}_M$ -chains describe how well the set of earthquake is representative of a single group in terms of their exhibited activities. This analysis may be useful to discover whether different earthquakes tended to indicate "syndromes" of some earthquake group. The corresponding $\widehat{C}_A-$ chains describe how well particular subsets of activities are representative of earthquakes as whole and hence could be used to identify patterns of activities which are strongly indicative of the earthquake. Speaking about connectivity we imply informational connection, which also may be physical.

3^0. Now consider measure of connectivity. Preliminarily adduce necessary definitions:

Definition 1. Consider a pair of n-dimensional vectors $\overrightarrow{a}$ and $\overrightarrow{b}$. The set of ordered pair

$$\{(a_i, b_i):\ \textit{at least } a_i \textit{ or } b_i > 0\}$$

is called the support of $\left\{(a_i, b_i) : i = \overline{1, n}\right\}$.

Definition 2. Two vectors $\overrightarrow{a}$ and $\overrightarrow{b}$ are said to be "equivalent" if all pairs (a_i, b_i) of support are such that $a_i = b_i$. Two equivalent vectors have a unit connectivity.

Definition 3. Two vectors $\overrightarrow{a}$ and $\overrightarrow{b}$ are said to have "absolute disparity" if in all pairs in the support of $\{(a_i, b_i) : i = \overline{1,n}\}$ exactly one of component a_i or b_i is zero. In addition two vectors are absolutely disparate if they have null support. Two absolutely disparate vectors have a zero connectivity.

Definition 4. For two vectors having elements $a_i, b_i \in [0, 1]$ for all i which are neither equivalent nor absolutely disparate the connectivity is given by proportion of pairs (a_i, b_i) in the support such that $a_i = b_i = 1$.

Let identify the measure of connectivity with the scalar product of vectors $\overrightarrow{a}$ and $\overrightarrow{b}$, $a_i * b_i$, which can be introduced proceeding from above definitions and commutativity of product. Notice that, all saying is related with both fuzzy and non-fuzzy vectors, but for non-fuzzy vectors one receive the unique expression:

$$C = \left(\sum_{i=1}^{n} (a_i \wedge b_i) \right) / \left(\sum_{i=1}^{n} (a_i \vee b_i) \right), \tag{1.16}$$

whilst for fuzzy vectors the expression of scalar product is no longer unique.

Let give a simple rule of apportionment of representative chains: in tables(2-6) such chains are fairly large and highly connected (excluding chains with one component). To each selected chain is attached the level of connectivity by definition 4 [1]. We receive the following table:

Table5

positive-connectivity		negative-connectivity	
value of connectivity	representative $+\hat{c}_{\mathcal{M}}^{(1)}-$ chains	value of connectivity	representative $-c_{\mathcal{M}}^{(1)}-$ chains
1		1	no
0,75	$\{\mathcal{M}_{u_1}^{(1)}, \mathcal{M}_{u_2}^{(1)}\}$	0,75	representative
0,5	$\{\mathcal{M}_{u_1}^{(1)}, \mathcal{M}_{u_2}^{(1)}, \mathcal{M}_{u_4}^{(1)}\}$	0,5	chains
0	$\{\mathcal{M}_{u_1}^{(1)}, \mathcal{M}_{u_2}^{(1)}, \mathcal{M}_{u_3}^{(1)}, \mathcal{M}_{u_4}^{(1)}\}$	0	
positive-connectivity		negative-connectivity	
value of connectivity	representative $\hat{c}_{\mathcal{A}}^{(1)}-$ chains	value of connectivity	representative $-c_{\mathcal{A}}^{(1)}-$ chains
1		1	no
0,75	$\{\mathcal{A}_1, \mathcal{A}_4\}$	0,75	representative
0,5	$\{\mathcal{A}_1, \mathcal{A}_3, \mathcal{A}_4\}$	0,5	chains
0	$\{\mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3, \mathcal{A}_4\}$	0	

[1] In the case of non-fuzzy data this rule gives a number of connected vertex divided by whole number.

Table6

positive-connectivity		negative-connectivity	
value of connectivity	representative $+\hat{C}_M^{(2)}-$ chains	value of connectivity	representative $-C_M^{(1)}-$ chains
1		1	
0,8		0,8	no
0,6	$\{M_{u_2}^{(2)}, M_{u_4}^{(2)}\}$	0,6	representative
0,4	$\{M_{u_1}^{(2)}, M_{u_2}^{(2)}, M_{u_3}^{(2)}, M_{u_4}^{(2)}, M_{u_5}^{(2)}\}$	0	chains
0	$\{M_{u_1}^{(2)}, M_{u_2}^{(2)}, M_{u_3}^{(2)}, M_{u_4}^{(2)}, M_{u_5}^{(2)}\}$	0	

positive-connectivity		negative-connectivity	
value of connectivity	representative $+C_A^{(2)}-$ chains	value of connectivity	representative $-\hat{C}_A^{(2)}-$ chains
1		1	no
0,75	$\{A_1, A_4\}$	0,75	representative
0,5	$\{A_1, A_3, A_4\}$	0,5	chains
0	$\{A_1, A_2, A_3, A_4\}$	0	

Table7

positive-connectivity		negative-connectivity	
value of connectivity	representative $+\hat{C}_M^{(3)}-$ chains	value of connectivity	representative $-C_M^{(3)}-$ chains
1		1	
0,8		0,8	no
0,6	$\{M_{v_1}^{(3)}, M_{v_3}^{(3)}\}$	0,6	representative
0,4	$\{M_{v_1}^{(3)}, M_{v_2}^{(3)}, M_{v_3}^{(3)}, M_{v_5}^{(3)}\}$	0	chains
0	$\{M_{v_1}^{(3)}, M_{v_2}^{(3)}, M_{v_3}^{(3)}, M_{v_4}^{(3)}, M_{v_5}^{(3)}\}$	0	

positive-connectivity		negative-connectivity	
value of connectivity	representative $+C_A^{(3)}-$ chains	value of connectivity	representative $-\hat{C}_A^{(3)}-$ chains
1		1	no
0,75	$\{A_2, A_3, A_4\}$	0,75	representative
0,5	$\{A_2, A_3, A_4\}$	0,5	chains
0	$\{A_1, A_2, A_3, A_4\}$	0	

Tables 4-7 establish the distribution of connectivities by representative chains. We especially were interested in uncertainty distribution by representative chains of activities. For any certain image of activities possibility distribution by earthquake powers is resulting:

$$\delta\left(M^{(j)}\right) = \frac{1}{2}\left(\chi_{Large}\left(\frac{\sum_i C_i P\left(Qi \mid \{A^{(j)}\}\right)}{\sum_i C_i}\right.\right.$$

$$+\chi_{Small}\left(\frac{\sum_k d_k P\left(R_k \mid \left\{A^{(j)}\right\}\right)}{\sum_k d_k}\right)\Bigg), \tag{1.17}$$

where $M^{(j)}$ indicates the group of earthquake, C_i is the measure of connectivity corresponding to the chain Qi, $P\left(Qi \mid \left\{A^{(j)}\right\}\right)$ denotes the proportion of activities in Qi which are presented in $\left\{A^{(j)}\right\}$, $P\left(R_k \mid \left\{A^{(j)}\right\}\right)$ is corresponding proportion in the case of negative connectivity, ,,Large" and ,,Small" are fuzzy subsets of the interval $[0;1]$.

Let for example$\left\{A^{(j)}\right\} = \{A_1, A_3, A_4\}$. Using tables 4-7 and (1.17) after simple calculations one receive:

$$\begin{aligned}\delta\left(M^{(1)}\right) &= \frac{1}{2}\left(\chi_{Large}(0.7)+1\right), \delta\left(M^{(2)}\right) = \frac{1}{2}\left(\chi_{Large}(1)+1\right), \delta\left(M^{(3)}\right) \\ &= \frac{1}{2}\left(\chi_{Large}\left(\frac{1}{3}\right)+1\right)\end{aligned}$$

Independently of concrete form of χ_{Large} it is evident that the decision is $M^{(2)}$. But in more general case the form of $\chi_{Large}$and χ_{Small} is very important.

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