MATHEMATICAL MODEL OF OPTIMAL LONGTERM FUNCTIONING OF THE ENERGY SYSTEM

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Abstract

A mathematical model of optimal longterm functioning of the energy system of the region is suggested. The model is of a mathematical programming type. The minimum of total annual expenditures or the minimum of the weighted sum of the shortages of the electric energy is taken as a criterion of optimum.

Key words and phrases: Series, rearrangement, almost everywhere convergence, normed space, Orlicz space, Rademacher functions, mathematical expectation.

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1. *Introduction*

During the work of the energy system, including various types of power stations, there arises a problem of optimal functioning of the system. The functioning of a system is a synchronous work of all energy blocks. In fact, it means a regulation of storage of water and fuel. The work of energy system is considered to be optimal, if for a given (determined or random) temporary schedule of demand the criterion of optimum reaches its extreme value. Usually the minimum of total expenditure or the maximum of the produced electric energy is taken as a criterion of optimum.

There are many scientific works dedicated to the optimal regimes of electric systems and separate energy blocks. In particular, mathematical models composed by means of dynamic programming are most remarkable [2,3,4,7]. They are very extensive - provided that they can take into account all technical requirements of the problem and can be numerically realized for practically any producing function of an arbitrary analytical nature. However, these models are not practically useful for the systems which contain more than two hydroelectric stations.

In this light models of mathematical programming are more useful. Such models are used both for projecting and composing shortterm and longterm work schedules of the energy systems. There are various models of this type [1,5,6,8,9,10], some of them include the whole fuel-energy complex, projecting of the electric transmission lines, etc., but all of them do have one common fault - either the factor of optimal regulation of storage of water and fuel is not taken into account [1,5,6,10], or this factor does not participate in the mathematical model [8,9].

Suggested mathematical model is devoid of the faults noted above and in our opinion it describes the posed problem well enough.

Let us start to compose the mathematical model. For this purpose let's enumerate the parameters, which practically define the posed problem. Note first of all, that due to the periodicity of the water discharge it is natural to take one year as a key horizon of planning. The horizon of planning is divided into periods. Usually the cycles consisting of one week (52 periods), two weeks (26 periods), one month (12 periods) and of one quarter (4 periods) are considered. Let us consider generally N periods for one year, n hydroelectric power stations, m thermal power stations and the following index: $t = 1, 2, \ldots, N$; $i = 1, 2, \ldots, n$; $j = 1, 2, \ldots, m$. Let for the period t r_t be the demanded electric energy, ξ_{it} be the flow of the water from the river into the water reservoir of i-th hydroelectric station (determined or a random variable with the given distribution). Besides we are given the limited capacities N_i and M_i of the hydro and thermal stations respectively, lower and upper limits v_i and V_i of the volume of water in the i-th reservoir, the quantity of the provided fuel K_j for the j-th thermal station per year, limited passing capacity P of the transmission lines for exporting and importing the electric energy.

For the convenience we shall be measuring all the variables, including the quantities of water and fuel with the equivalent quantities of electric energy.

Let us introduce the following notations:

x_{it}–water discharge from the i-th reservoir into the turbines during the period t;

η_{it}–overflow of unutilized water from the i-th reservoir during the period t;

y_{jt}–quantity of fuel, utilized by the j-th thermal station during the period t;

z_t^+ and z_t^-–sold and purchased energy respectively during the period t;

a_i and b_j–expenses for producing the unit of the electric energy for the i-th hydroelectric and j-th thermal station respectively;

d_t–a penalty for lack of the unit of electric energy for the period t;

g–a penalty for the overflow of the unit of unutilized water;

c_t^+ and c_t^-–the cost of the unit of sold and purchased electric energy respectively for the period t.

$x_{it},\ \eta_{it},\ y_{jt},\ z_t^+,\ z_t^-$ $(i=\overline{1,n};\ j=\overline{1,m};\ t=\overline{1,N})$ are the desired quantities which have to satisfy the conditions of the problem. First of all they must obey these two-sided constraints

$$\begin{aligned} &0 \le x_{it} \le N_i, \qquad \forall i,t;\\ &0 \le y_{jt} \le M_j, \qquad \forall j,t;\\ &0 \le z_t^+ \le P, \qquad \forall t;\\ &0 \le z_t^- \le P, \qquad \forall t\,. \end{aligned} \tag{1.1}$$

Here below we bring constraints which express the substance of the problem: constraints for the usage of fuel

$$\sum_{t=1}^{N} y_{jt} \le K_j, \quad \forall j\,; \tag{1.2}$$

constraints related to the capacities of the reservoirs

$$v_i \le v_i^0 + \sum_{t=1}^{\tau}(\xi_{it} - x_{it} - \eta_{it}) \le V_i, \quad \forall i, \tau = \overline{1,N}\,; \tag{1.3}$$

constraints that satisfy the demand

$$\sum_{i=1}^{n} x_{it} + \sum_{j=1}^{m} y_{jt} - z_t^+ + z_t^- \le r_t, \quad \forall t\,. \tag{1.4}$$

If the system contains cascade hydroelectric stations with numbers $l, l+1, \dots$ (numbering downwards), then for the l-th station (reservoir) constraint (1.3) remains valid and for the rest of them it will have a rather different form

$$v_{l+k} \le v_{l+k}^0 + \textstyle\sum_{t=1}^{r}(x_{l+k-1,t} + \eta_{l+k-1,t} - x_{l+k,t} - \eta_{l+k,t} + \zeta_{l+k,t}) \le V_{l+k},$$
$$\forall \tau,\ k = 1,2,\dots \tag{1.5}$$

where v_i^0 is an initial capacity of water in the i-th reservoir and $\zeta_{l+k,t}$ is an additional inflow on the section $(l+k-1, l+k)$ during the period t, if such inflow exists.

As a whole, in the energy system a specific correlation among the peak and base electric energies has to be fulfilled on every stage. If z_t^- base energy is purchased and z_t^+ peak energy is sold, then we express it as follows:

$$\alpha_t \le \frac{\sum\limits_{i=1}^{n} x_{it} - z_t^+}{\sum\limits_{j=1}^{m} y_{jt} - z_t^-} \le \beta_t\,. \tag{1.6}$$

These are all the conditions and constraints to which the labour of the system must be subjected. In this connection the variables $x_{it}, y_{jt}, \eta_{it}, z_t^+, z_t^-$ have to be chosen in order to achieve the best effect.

As the criterion of efficiency we take a minimum of the summarized expenditures. The summarized expenditures per year are expressed as follows:

$$F(x_{it}, y_{jt}, \eta_{it}, z_t^+, z_t^-) = \sum_{i=1}^{n}\sum_{t=1}^{N}(a_i x_{it} + g\eta_{it})$$

$$+\sum_{j=1}^{m}\sum_{t=1}^{N} b_j y_{jt} + + \sum_{t=1}^{N} c_t^- z_t^- - \sum_{t=1}^{N} c_t^+ z_t^+ \tag{1.7}$$

$$+\sum_{t=1}^{N} d_t(r_t - \sum_{i=1}^{n} x_{it} - \sum_{j=1}^{m} y_{jt} + z_t^+ - z_t^-)\,.$$

The last member in (1.7) expresses a weighted annual shortage.

As another criterion of optimum we consider the functional:

$$F(x_{it}, y_{jt}, \eta_{it}, z_t^+, z_t^-) =$$

$$\sum_{t=1}^{N} d_t(r_t - \sum_{i=1}^{n} x_{it} - \sum_{j=1}^{m} y_{jt} + z_t^+ - z_t^-) + \sum_{i=1}^{n}\sum_{t=1}^{N} g_i \eta_{it}\,. \tag{1.8}$$

The significance of the last member in (1.8) will be discussed below. Note, that the relation among the coefficients in (1.7) and (1.8) are formally represented as follows:

$$d >> c^+ > c^- > b > a >> g\,. \tag{1.9}$$

Thus we received the following problem of mathematical programming: find the values of variables x_{it}, y_{jt}, η_{it}, z_t^+, z_t^- satisfying the constraints (1.1)–(1.6) and minimizing the functional (1.7) or (1.8).

Let us finally make some remarks:

– since the parameters ξ_{it}, ζ_{lt}, r_t are the random variables, we deal with the problem of stochastic programming. It is natural to use a method of sequential correction to solve this problem: first we solve the determined problem – the random parameters are replaced by the mean values of statistical data of the previous years. In case of the significant discrepancies between the virtual and planning values of random variables, we correct the planning according to the newly forecasted data;

– as we have mentioned above, the quantity of water is measured by the units of electric energy. Here an obstacle appears. The connection between the quantity of electric energy X and the expenditure of water Q is given by the formula $X = 9,81\eta HQ$ [8], where H is a full pressure of water in the turbine and η is a coefficient of efficiency of the aggregates of the electric power station. But in the process of work of the station Q and H are correlated and this correlation, in general, is complicated and is different for various electric stations with water reservoirs. There is only one way out – taking mean values per quarter (or per month, if possible) as H. In this case we deal with the model of linear programming;

– the constraints (1.6) are written for the case, when base energy is purchased and peak is sold. Otherwise (1.6) can be easily altered.

– a few words about an inequality (1.4). It is accepted to write an inverse inequality or equation. The inverse of (1.4) seems unreal to us, and the equation, if the situation permits, would, of course, reflect exactly what we desire. For the regions with the lack of electric energy the constraints ought to have the form (1.4), besides it will be suitable for other cases since they will be transformed in the equation due to the condition (9);

– the variables η_{il} are fictitious. However small is the coefficient g, the inequality (1.3) will be automatically fulfilled and g must be taken as small as to neglect its influence on the value of objective function.

The model, suggested here, has been applied to the energy system of Georgia. As a horizon of planning was taken one year, divided into 12 periods (months). The periods were enumerated from April. As a flow of water into the reservoir mean representative samples per month, according to the statistical data of previous years, and forecasted data for the demand r_t were taken. As the full pressure H_i in the turbines for each hydrostation with reservoir average annual pressure was taken. Thus the problem has been solved according to the standard schema of linear programming. The minimum of total expenditures served as the optimal criterion. The balances of electric energy (1.4) were taken into account separately for basic and for peak energies for each month – in the winter period the basic made up 60% and in the summer period – 40% of the consumable energy. The system consisted of one thermal and 27 hydroelectric power stations, 13 among them with water reservoirs.

Two variants were computed by IBM PC AT. For the first one the system was considered closed – it was not allowed to sell or to purchase electric energy. For another it was permitted to purchase basic and to sell peak energy (in addition the ratio of c_t^- with c_t^+ was one half). In both variants the thermal stations and hydroelectric stations without reservoirs worked in a basic regime. Hydrostations with reservoirs in the first one worked partially in basic regime and in the second one – almost entirely in the peak regime. At the expense of selling of peak energy the second variant gives a significant economic effect. Moreover, it should be noted, that according to the obtained optimal schedules the waste of water η_{it} was observed only in the small reservoirs and in the initial periods.

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