# ANALYTIC AND NUMERICAL SOLUTIONS OF ATMOSPHERIC GRAVITY WAVES EVOLVING IN HORIZONTAL SHEAR FLOW

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*Abstract*

The evolution of atmospheric gravity waves (AGW) in unbounded horizontal winds with constant shear is developed for the three-dimensional case. The atmosphere is considered to be isothermally stratified and the wind shear lies in the horizontal plane. Time varying frequencies of the AGW, which for large times tend to the isothermal Brunt-Väisälä frequency is obtained. The excitation of vortical perturbations located in the horizontal plane – shear waves, is shown. The time variation of AGW frequencies, the existence of shear wave and their amplitudes amplification/damping are due to the presence of the horizontal shear flow. The horizontal shear flow as a possible source of formation of large scale travelling ionospheric disturbances (TIDs) with small period ($16-19min$) is considered. The different pictures of evolution of the AGW like initial perturbation are obtained in vertical, horizontal winds and its shear directions. The importance of coupling between the gravity and shear waves in the horizontal shear flow is shown. The possibility of transformation of the shear waves into the AGWs is demonstrated.

*Key words and phrases*:Hydrodynamic Equations; Atmospheric Gravity Waves; Shear Flow.

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## 1. *Introduction*

Shear flow is a possible energetic source for excitation of various types of waves in the atmosphere (see [1-4]). The different sources for the excitation of the atmospheric gravity waves (AGWs) are known (see [5-8]) and they essentially contribute to the energetics of the lower and upper atmosphere. In the earlier (see [9]) and more later considerations (see [10-13]) the wave mean flow interactions are noted to be essential for the AGWs generation and their propagation in the thermosphere. We consider the phenomenon of mean flow wave interaction as a possible source of the AGWs amplification in the thermosphere and also for the lower atmosphere.

In the present paper we develop the linear theory of evolution of initial atmospheric gravity wavelike perturbations in the horizontal shear flow for the three-dimensional case. Hereafter, under the horizontal shear flow we mean constant shear of horizontal winds located in the same plane. The atmosphere is considered to be invisced, isothermally stratified and velocity perturbation increases with the height as $\rho_0^{-\frac{1}{2}}$ (see [5]), where $\rho_0$ is the background density of the atmosphere. We use transformation of perturbations which corresponds to the nonmodal analysis in a moving coordinate frame. This method of change of independent variables from Cartesian to a moving frame is involved in early works of Lord Kelvin [14] and Orr [15]. Use of the spatial Fourrier expansion in the moving frame enables to change the spatial inhomogeneity in the governing equations with inhomogeneity in time. This type of nonmodal analysis of evolution of the initial wavelike perturbation in linear shear flow for two-demensional cases was recently developed by many authors: Farrell and Ioannou [16], Chagelishvili et al. [17] and Rogava and Mahajan [18]. Farrell and Ioannou [16] and Rogava and Mahajan [18] considered evolution of internal waves in horizontal winds with vertical constant shear.

In the present investigation, the horizontal shear flow gives different pictures of the evolution of initial wavelike perturbation, compared to the investigation of the cases of vertical shear, referred above. The three-dimensionality is essential – in any direction the pictures of evolution of initial wavelike perturbation of velocity are different. On the basis of modified exponential modal analysis (see [19,20]), for which wave frequency and its exponential amplification/damping rates depend on time, it is convenient to investigate evolution of wavelike initial perturbation.

Differently earlier works present investigation allows us to obtain:

1. Equation describing amplification/damping and coupling of the AGW and the vortical perturbation located in the horizontal plane – shear waves in horizontal shear flow;

2. The analytical approach to a time varying spectrum of the AGW and shear waves in the cases of their independent evolution in the horizontal shear flow; The possibility of transformation of the shear waves into the AGWs in the horizontal shear flow;

3. The large scale ionospheric disturbances (TIDs) with small (16-19min) periods as a manifestation of propagation of the AGW evolving in the horizontal shear flow.

The paper has the following structure: In section 2 a set of linear equations describing the evolution of AGW in horizontal shear flow is obtained.

In section 3 the set of general equations for certain atmospheric conditions is solved analytically. In section 4 the coupling between the AGW and shear waves is considered numerically. Concluding remarks are given in section 5.

## 2. *Mathematical formalism*

The internal waves in the ivisced atmosphere, when the gravity force and background horizontal shear flow are taken into account, are described by the following set of linearized equations:

$$\rho_0\left[\frac{D\vec{v}}{Dt}+(\vec{v}\cdot\nabla)\,\vec{u}_0\right]=-\nabla p+\rho\vec{g}, \qquad (1a)$$

$$\frac{D\rho}{Dt}+\vec{v}\cdot\nabla\rho_0+\rho_0\nabla\cdot\vec{v}=0, \qquad (1b)$$

$$\frac{Dp}{Dt}=-{c_s}^2\rho_0\nabla\cdot\vec{v}-\vec{v}\cdot\nabla p_0, \qquad (1c)$$

where

$$\frac{D}{Dt}=\frac{\partial}{\partial t}+\vec{u}_0\cdot\nabla,$$

$\vec{v}=\vec{v}(u,v,w)$ is the velocity; $u,\ v,\ w$ are the components of the perturbed velocity in $x$, $y$ and $z$ direction, respectively. $x$, $y$ are the horizontal and $z$ is the vertical coordinate; $\mathbf{g}$ is the acceleration of gravity; $\vec{u}_0=\vec{u}_0(u_0,0,0)$ is the basic state horizontal wind. For horizontal shear flow, $u_0$ is given by

$$u_0=ay, \qquad (2)$$

where $a$ is the shear of the horizontal wind; $\rho$ and $p$ are the perturbations of density and pressure, respectively; $\rho_0$, $p_0$ are the unperturbed values; $c_s$=$(\gamma\frac{p_0}{\rho_0})^{1/2}$ is the velocity of sound , $\gamma$ is the ratio of specific heats. In (1a) the condition of hydrostatic equilibrium,

$$\rho_0\mathbf{g}-\nabla\mathbf{p_0}=\mathbf{0}, \qquad (3)$$

has been removed. For an isothermal atmosphere, equation (3) gives the well known barometric height profile for the unperturbed (background) pressure and density:

$$p_0=p_{00}exp\left(-\frac{z}{H}\right), \qquad \rho_0=\rho_{00}exp\left(-\frac{z}{H}\right), \qquad (4)$$

where $H=c^2/\gamma g$ is the atmospheric scale height, $p_{00}$ and $\rho_{00}$ are the pressure and density at height $z=0$, respectively. Sound velocity $c_s$ and atmospheric scale height $H$ is assumed as constant with height.

We introduce new variables:

$$x^{(1)} = x - ayt; \quad y^{(1)} = y; \quad z^{(1)} = z \tag{5}$$

and transformed atmospheric parameters by means of the following relations:

$$p, \rho(x, y, z, t) = p_1, \rho_1(x^{(1)}, y^{(1)}, z^{(1)}, t) \cdot exp\left(-\frac{z^{(1)}}{2H}\right), \tag{6}$$

$$u, v, w(x, y, z, t) = u_1, v_1, w_1(x^{(1)}, y^{(1)}, z^{(1)}, t) \cdot exp\left(\frac{z^{(1)}}{2H}\right). \tag{7}$$

This well known transformation (5) effectively takes us from a Cartesian to the local rest frame of the basic flow (see [6,21]). The relations (6), (7) are convenient for describing AGW (see [5]). In the new variables, (5), for the disturbed parameters, (6) and (7), the Fourier transformation is used:

$$q_1 = \int_{-\infty}^{+\infty} q_k(t) exp[i(k_x x + (k_y - ak_x t)\, y + k_z z)] dk_x dk_y dk_z =$$

$$\int_{-\infty}^{+\infty} q_k(t) exp\Big[i(k_x x^{(1)} + k_y y^{(1)} + k_z z^{(1)})\Big] dk_x dk_y dk_z, \tag{8}$$

where $q_1 \equiv \{p_1, \rho_1, u_1, v_1, w_1\}$ and $q_k(t) \equiv \{p_k, \rho_k, u_k, v_k, w_k\}(t)$ are the vectors of basic perturbed values and their spatial Fourier harmonics (SFH), respectively; $\vec{k} = \vec{k}(k_x, k_y, k_z)$ is the wavenumber.

Use of (5)-(8) in equations (1), for the amplitudes $q_k(t)$ gives the following equations

$$\rho_{00} u_k^{'} = -ik_x p_k - a\rho_{00} v_k, \tag{9a}$$

$$\rho_{00} v_k^{'} = -ik_t p_k, \tag{9b}$$

$$\rho_{00} w_k^{'} = -\left(ik_z - \frac{1}{2H}\right) p_k - g\rho_k, \tag{9c}$$

$$\frac{\rho_k^{'}}{\rho_{00}} = -i\left(k_x u_k + k_t v_k\right) - \left(ik_z - \frac{1}{2H}\right) w_k, \tag{9d}$$

$$\frac{p_k^{'}}{\rho_{00} c_s^2} = -i\left(k_x u_k + k_t v_k\right) - \left(ik_z - E\right) w_k, \tag{9e}$$

$$k_t(t) = k_y - ak_x t, \tag{9f}$$

where $E = \frac{2-\gamma}{2\gamma H}$ is the isothermal Eckart coefficient (see [7]). Here and hereafter $(') \equiv \frac{\partial()}{\partial t}$.

Thus $k_t(t)$, in linear approximation, displays a drift of the Fourier harmonics in the $k$-space. Equations (9) make up a mathematically closed system of differential equations. These linearized equations, in the absence of shear flow, describe acoustic- gravity waves in invisced isothermal atmosphere (see [5]).

In order to consider evolution of the SFH of small perturbation described by equations (9) the following characteristic time is used

$$t_a = 2\left|\frac{k_y}{ak_x}\right|, \tag{10}$$

which in these equations characterizes variation of $k_t(t)$ (equation (9f)) and the time inhomogeneity, respectively. Hereafter we consider shear $a > 0$. Time intervals of $\Delta t \ll t_a$ correspond to the small changes of wavenumber $k_t(t)$. In the present paper the following analysis will show that description of evolution of the initial perturbation in horizontal shear flow by means of time $t_a$ is convenient. In the cases when the period of plane wavelike perturbations is sufficiently small in comparison with $t_a$ one usually expects that the set of linear equations (9) describes the quasisinusoidal oscillations, and the spectrum corresponds to the AGW obtained by Hines [5].

## 3. *The existence of atmospheric gravity and shear waves evolving in horizontal shear flow*

It should be noted that in the governing equations (1) the inhomogeneity with space is caused by the horizontal winds ($u_0(y)$), unperturbed density ($\rho_0(z)$) and pressure ($p_0(z)$). Use of transformation (5)-(8) allows us to reduce these governing equations to the set of equations (9), where inhomogeneity ($k_t(t)$) occurs in time. Thus, we have reduced the set of equations (1) to the initial-value problem. In order to obtain the spectrum of the AGWs in the presence of shear flow we will transform the set of equations (9) to the more easily analyzed form. Taking the derivatives of equation (9d) with respect to time and eliminating $u_k$, $v_k$ and $w_k$ using of (9a)-(9c) and (9e) allows us to obtain the following equation in the case of $|k_z| > \frac{1}{2H}$:

$$\left(k_x^2 + k_t^2 + k_z^2 + \frac{1}{4H^2}\right)\rho_k^{'''} + \omega_B^2\left(k_x^2 + k_t^2\right)\rho_k^{'} - $$
$$4ak_xk_t\left[\omega_B^2\rho_k + \rho_k^{''}\right] = 0, \tag{11}$$

where $\omega_B = \left[\frac{(\gamma-1)H}{\gamma g}\right]^{1/2}$ is the isothermal Brunt-Väisälä frequency. The last term in (11) is caused by the presence of the horizontal shear flow.

We have considered longperiod approach and neglected higher orders of time derivatives in (11). In (11) for large times of $t \gg t_a$ coefficients of $\rho_k^{'''}$ and $\rho_k^{'}$ are proportional to $t^2$ and the coefficient $(4ak_xk_t)$ of the last term $(\omega_B^2\rho_k + \rho_k^{''})$ is proportional to $t$. These dependences show that in equation (11) for large times of $t \gg t_a$ the last term is comparably small. Therefore integrating with respect to the time we obtain equation which describes oscillations of the SFH amplitudes of wavelike perturbation by the isothermal Brunt-Väisälä frequency, $\omega_B$. So, it can be said that for great times of $t \gg t_a$ equation (11) describes oscillations of the AGWs with Brunt-Väisälä frequency and with amplitude which is nearly saturated.

We seek an analytic solution of (11) in the following form:

$$\rho_k = A \cdot exp\left[-i\Omega(t)\right], \tag{12}$$

where $\Omega(t) = \Omega_r(t) + i\Omega_i(t)$ is the complex valued phase, $\Omega_r(t)$ and $\Omega_i(t)$ are the real and imaginary parts of the phase, respectively. $\Omega_r^{'}(t)$ and $\Omega_i^{'}(t)$ are the time dependent frequency of waves and its corresponding perturbed density amplitudes amplification/damping rate, respectively. When $\Omega^{'}$ do not depend on time, equation (12) corresponds to the usual exponential modal transformation, which in the case of no shear, $a = 0$, from equations (9) gives the spectrum of AGW (see [5]) in invisced isothermal atmosphere. Transformations (8) and (12) correspond to the modified exponential modal transformation, which is convenient to investigate the spectrum of in shear flow evolving waves. According to transformation (12) value $A \cdot exp\left[\Omega_i(t)\right]$ is the time varying amplitude of $\rho_k$.

If we consider the small changes of phase

$$\left|\frac{\Omega''}{\Omega'}\right| \ll \omega_B, \tag{13}$$

use of (12) in (11) for $\Omega^{'}$ gives the following solution:

$$\Omega_{1,2}^{'} = \pm\omega_g(t) + i\Omega_{ig}^{'}(t), \tag{14}$$

where value $\omega_g(t)$

$$\omega_g^2(t) = \frac{\omega_B^2\left(k_x^2 + k_t^2\right)}{k_x^2 + k_t^2 + k_z^2 + \frac{1}{4H^2}}, \tag{15}$$

is the time varying frequency of atmospheric gravity waves. In the case of no shear, $a = 0$, this expression does not depend on the time and corresponds to the frequencies of the large wavelength, $\lambda_x, \lambda_y, \lambda_z > H$, AGWs given in Refs [22-24]. Here $\lambda_x$, $\lambda_y$ and $\lambda_z$ are wavelengths in horizontal $x$, $y$

and vertical $z$ direction, respectively. The amplification/damping rate of gravity waves, $\Omega'_{ig}$, is:

$$\Omega'_{ig}(t) = -\frac{3\omega'_g}{2\omega_g} + \frac{2ak_xk_t}{k_x^2 + k_t^2 + k_z^2 + \frac{1}{4H^2}}\left[1 - \frac{\omega_B^2}{\omega_g^2(t)}\right]. \tag{16}$$

In (15) and (16) condition (13) is identical to inequality $\omega_g \gg \left|\Omega'_{ig}\right|$ which has been used.

Values $\omega_g(t)$, (15), and $\omega_{ig}(t)$, (16), describe the evolution of atmospheric gravity waves in the horizontal shear flow. For the perturbed densities SFH amplitude of the AGW using equations (15) and (16), from (12) we obtain the following expression:

$$\rho_{kg} = A_g\left(\frac{\omega_g}{\omega_{g0}}\right)^{\frac{1}{2}} exp\left[-i\Omega_g(t)\right], \tag{17}$$

where $\Omega_g(t) = \int_0^t \omega_g\left(t'\right)dt'$, $\omega_{g0} \equiv \omega_g(t=0) = \omega_g(\alpha=0)$ and $A_g$ implies the amplitude of the SFH of gravity waves at initial time $t = 0$. Here and hereafter for the simplicity the initial condition $\Omega(t=0) = 0$ is used.

According to equation (15) the gravity wave frequency,$\omega_g(t)$, has the minimum values at time $t = \frac{t_a}{2}$ and for large times of $t \gg t_a$ tends to the isothermal Brunt-Väisälä frequency $\omega_B$. So, the isothermal Brunt-Väisälä frequency $\omega_B$ is cutoff frequency of the in horizontal shear flow evolving AGWs. Equations (15)-(17) also show that the increase of amplitudes of the SFH of density perturbation of the AGW appears at time $t > t_a$. So, solution (14) of equation (11) describes the formation of the AGWs with small periods (nearly $\frac{2\pi}{\omega_B}$). According to equation (15) and (16) for large times of $t \gg t_a$the phase velocity of the gravity waves in $x$ direction tends to the value of $\frac{\omega_B}{k_x}$.

Henceforth in estimations, for the terrestrial atmosphere the value $\gamma = 1.4$ will be used ([7]). In the following we choose the values of wavenumber in accordance with condition $\left|\frac{k_z}{k_{x,y}}\right| = \frac{\lambda_{x,y}}{\lambda_z} = 3 - 10$. The ionosphere manifestation of the AGWs propagation is TIDs (see [5], [13] and [25]). If we choose the wavelengths in accordance with the sizes of the large scale TIDs, $\lambda_x \geq 1000km$, atmospheric scale height $H = 60km$ and winds direction, $x$, is the meridional and its shear has the zonal direction, then for times of $t > t_a$ the gravity wave period tends to the period about $15 - 16min$. This case corresponds to the formation of large scale TIDs with small period and phase velocity ($\simeq \frac{\omega_B}{k_x}$) in the meridional direction about $\geq 1000 - 1100m \cdot s^{-1}$. The period of nearly $16 - 19min$ is in agreement with observation of large scale TIDs by Williams et al. [26], Bowman

[27] and also observations by Oya et al. [28]. The formation of the small period AGW (nearly Brunt-Väisälä period) in the horizontal shear flow will be more clearly illustrated numerically in the next section.

Note, that the molecular viscosity, which is essential for the dissipation of gravity waves in the ionosphere F-region (see [5]), for the large scale gravity waves, $\lambda_{x,y} \gg H$, is comparably small (see [29,30]) and in equations (9) for simplicity we have neglected it. According to Mayr et al. [10] and Richmond [30] for gravity wave with a horizontal phase velocity about 750 $m\ s^{-1}$ the temperature and wind perturbations at thermospheric heights are decaying slowly before reaching the equator. So one may expect that gravity wave can possibly appear at the mid-latitude F-region. In this case we consider the polar regions as the main source of the AGW generation of which the ionospheric manifestation is the large scale TIDs. In the present consideration the presence of horizontal shear flow is possibly an additional source of gravity wave generation and it would be the reason for the large scale TIDs observed in the mid-latitudes ionosphere.

Equation (11) has also the third solution with zero frequency ($Re(\Omega_3^{'}) = 0$) and the amplification/damping rate ( $Im(\Omega_3^{'}) \equiv \Omega_{ish}^{'}$) can be written as:

$$\Omega_{ish}^{'}(t) = \frac{4ak_x k_t}{k_x^2 + k_t^2}. \tag{18}$$

This type of wave (which is due to the presence of the horizontal shear flow) was referred as the shear wave (see [19]). Here and hereafter the subscriptions $g$ and $sh$ denote the values for gravity and shear waves, respectively. Using of (18) in (9) for the values of amplitudes of the SFH of shear waves we obtain the following expressions:

$$\rho_{ksh} = aA_{sh}\left(\frac{k_x^2 + k_y^2}{k_x^2 + k_t^2}\right)^2, \tag{19a}$$

$$p_{ksh} = \frac{aA_{sh}g\left(ik_z + \frac{1}{2H}\right)}{k_z^2 + \frac{1}{4H^2}}\left(\frac{k_x^2 + k_y^2}{k_x^2 + k_t^2}\right)^2, \tag{19b}$$

$$u_{ksh} = -\frac{A_{sh}g\left(k_z - \frac{i}{2H}\right)\left(k_x^2 + k_y^2\right)^2}{2\rho_{00}k_x^2\left(k_z^2 + \frac{1}{4H^2}\right)}\frac{k_t}{k_x^2 + k_t^2}, \tag{19c}$$

$$v_{ksh} = \frac{A_{sh}g\left(k_z - \frac{i}{2H}\right)\left(k_x^2 + k_y^2\right)^2}{2\rho_{00}k_x\left(k_z^2 + \frac{1}{4H^2}\right)}\frac{1}{k_x^2 + k_t^2}, \tag{19d}$$

$$w_{ksh} = 0, \tag{19e}$$

where $aA_{sh}$ corresponds to the amplitude of density perturbation for the shear waves at initial time $t = 0$. According to equations (19) changes of the amplitudes of the SFH of shear waves (which is due to the presence of horizontal shear flow) are essential at time $t \leq t_a$ and when $t \gg t_a$ the amplitudes vanish. According to equations (19) the shear wave corresponds to the standing type ($Re(\Omega') = 0$) incompressible ($\nabla \cdot \mathbf{v} = \mathbf{0}$) vortical ($(\nabla \times \mathbf{v})_z = const$) perturbation, excited in horizontal shear flow. The part of term $a\rho_{00}v_k$ in (9a) is similar to the effect of the Coriolis force for the traditional approach when the vertical component of this force is neglected. In this case vortical perturbation is amplified during the established balance between the perturbation of the shear direction velocity of the rotation driving background flow and the pressure gradient. In this case, according to equations (19) the amplification/damping of the amplitudes of the shear waves are essential at time $t \leq t_a$. According to equation (7) the amplitude of this type of atmospheric vortical perturbation – shear waves increase with height as $\sim \rho_0^{-\frac{1}{2}}$ $(\sim exp(\frac{z}{2H}))$. The last phenomenon and the essential dependence on the vertical wavenumber (see equation (19)), $k_z$, of the values of amplitude of the shear waves show its difference from the vortical waves considered in Ref. [31] for the case of incompressible fluids. One more essential dependence on $k_z$ for the shear waves amplitudes, which shows possibility of its transformation into the AGWs, will be considered in the next section. The excitation of the shear waves in horizontal shear flow is noticeable for three-dimensional case of evolution of wave-like perturbation. The effect of three-dimensionality of evolution of the AGW and shear waves more clearly appears in the numerical analysis of the evolution of amplitude of these waves in the horizontal shear flow. The effect of three-dimensionality of perturbation evolution in horizontal (parallel) shear flow for incompressible fluid without stratification was noted in Ref. [32].

Expressions (15)-(18) and (19) of independent evolution of the SFH amplitude of the AGW and shear waves in horizontal shear flow can be applied to small time intervals of $\Delta t < t_a$. In this case evolution of the AGW and shear waves can be considered independently. In this approach the possible changes of the amplitudes of SFH, caused by the coupling between gravity and shear waves in shear flow, are not taken into account. In the sequel we consider numerically the case of the evolving gravity and shear waves in horizontal shear flow for long time intervals of $\Delta t > t_a$, in which appearance of both type of waves' is essential.

## 4. *Coupling between the atmospheric gravity and shear waves*

The set of equations (9) describe evolution of the SFH amplitudes of coupled waves in stratified atmosphere in the presence of horizontal shear flow. equation (11) clearly shows the coupling between the AGW and vortical type perturbation – shear waves. Solutions (13)-(16) and (19) correspond to the case of independent evolution of the AGW and shear waves in the horizontal shear flow. In this case the changes of their amplitudes caused by the coupling between these waves are not taken into account. When the coupling between the AGW and shear waves is essential the values of amplification/damping rates of amplitudes of SFH of the AGW, (16), and that of the shear waves, (18), are not valid. When the changes of amplitudes of the AGW and shear waves due to the coupling in horizontal shear flow is essential then for the amplification/damping rates of gravity and shear waves the following dependence $\Omega'_{ig} = \Omega'_{ig}(\omega_g, \Omega'_{ish})$, $\Omega'_{ish} = \Omega'_{ish}(\omega_g, \Omega'_{ish})$ should be taken into account. According to these dependences the energy exchange between gravity and shear waves occured during characteristic time of excitation of shear waves,$t \leq t_a$, as well as during one period of the AGW.

Below we consider numerically the possibility of independent evolution of the AGW and shear waves and the changes of their amplitudes due to the coupling between these waves . For this we choose the initial condition in accordance with solution (15)-(17) for the describing evolution of the amplitude of SFH of the AGW and in accordance with equations (19) for the shear waves. In these cases to solve numerically the basic set of equations (9) of evolution of the amplitudes of the SFH of waves we use the following normalized values:

$$Q_K \equiv \{P_K, R_K, U_K, V_K, W_K\}(T) = \left\{ \frac{p_k}{\rho_{00} c_s^2}, \frac{\rho_k}{\rho_{00}}, \frac{u_k}{c_s}, \frac{v_k}{c_s}, \frac{w_k}{c_s} \right\}(t), \quad (20)$$

$$T = \omega_B t, \quad (21)$$

$$\vec{K}(K_x, K_y, K_z) = \vec{k}(k_x, k_y, k_z) \cdot 2H, \quad (22)$$

$$K_T(T) = K_y - \alpha K_x T. \quad (23)$$

Here $\alpha$ is the dimensionless shear

$$\alpha = \frac{a}{\omega_B}. \quad (24)$$

Let us note that according to (24) value

$$\alpha^{-2} = \frac{\omega_B^2}{\left(\frac{\partial u_0}{\partial y}\right)^2},$$

is the analogy of the Richardson number ($Ri = \frac{\omega_B^2}{\left(\frac{\partial u_0}{\partial z}\right)^2}$) for the horizontal linear shear flow. So, $\alpha^{-2}$ is a measure of the relative strength of the stratification and shear of background flow.

If we use those normalized values, (20)-(24), in equation (11) we obtain the following coefficients: $K_x^2 + K_T^2 + K_z^2 + 1$ of $\partial^3 R_K/\partial T^3$, $K_x^2 + K_T^2$ of $\partial R_K/\partial T$ and $4\alpha K_x K_T$ of $R_K^2 + \partial^2 R_K/\partial T^2$. When $K_x^2 + K_T^2 + K_z^2 + 1 \gg \left(K_x^2 + K_T^2, 4\alpha K_x K_T\right)$ the part of gravity waves in the spectrum of atmospheric waves evolving in horizontal shear flow should be essential. According to Didebulidze [19] the set of equations (9) describes evolution of the AGWs and shear waves as well as the high frequencies acoustic waves. For the cases $|K_x|, |K_y|, |K_z| = O(1)$the above referred coefficients in (11) are the same order for the normalized shear parameters $\alpha \simeq 0.25$. In these cases coupling between all types of waves (acoustic, gravity and shear waves) described by the set of equations (9) should be essential. Below we use values of shear parameters of $\alpha < 0.25$. In these cases we consider the coupling and possible independent evolution of the AGWs and shear waves in the horizontal shear flow by numerical solution of equations (9). Note that we have not considered the comparable small part of changes of amplitudes of the AGW due to the coupling with the acoustic waves in stratified atmosphere (see [9,23]).

Below we choose the values of horizontal winds shear, $a$, in the ionosphere F2-region in accordance with the review of the satelite wind measurements given in Mayr et al. [10]. According to Mayr et al. [10] the horizontal thermospheric wind measured in the polar regions is $800 - 2000\ m\ s^{-1}$ and more on magnetically disturbed days. In some cases the changes of this value of velocity for $2 - 5^o$ of latitude is sometimes of the same order as the value. Thus, it is possible to choose the horizontal shear of the horizontal winds as $a = 0.0001 - 0.001\ s^{-1}$, and even more. According to (24) these values of shear for the atmospheric scale height $H = 60km$ at ionosphere F2-region for the normalized shear correspond to $\alpha = 0.015 - 0.15$.

In Fig. 1 the evolution of the amplitude of SFH of normalized perturbed velocities $U_K(T)$, case (a), $V_K(T)$, case (b), and $W_K(T)$, case (c) of the AGW are plotted . In Fig. 2 that of the evolution of shear waves are plotted. For both figures the shear parameter $\alpha = 0.017$ and normalized wavenumbers $K_x = 0.3,\ K_y = 0.3,\ K_z = 1$. In these figures only the real parts of velocity amplitudes are plotted. For the shortest the numerical results of the evolution of perturbed density $R_K(T)$ and pressure $P_K(T)$ by their similarity of evolution $W_K(T)$ (cases (c) in Figs 1 and 2) are not plotted. The difference between evolution of gravity and shear waves is clearly seen from Figs 1 and 2. In both cases the changes of period and amplitudes of the AGW and shear waves due to their mutual coupling and the coupling with the acoustic waves are comparably small and their evolution pictures in the horizontal shear flow are in agreement with equations (15)-(19). In the cases demonstrated in Fig. 1 (case (c)) for the gravity wave, the sufficient increase of the SFH amplitudes of the vertical velocity is essential at time $T > T_a$. The evolution of the amplitude of SFH of the AGW in the horizontal shear flow (see Fig. 1) is characterized by the effect of three-dimensionality. The evolution of those amplitudes in wind's direction (see Fig. 1(a)), its shear direction (see Fig. 1(b)) and in the vertical direction (Fig. 1(c)) are different. According to the case demonstrated in Fig. 1, for large times of $T > T_a$ the AGW period tends to the isothermal Brunt-Väisälä period – which agrees to the solution, (14)-(16), of the independent evolution of gravity and shear waves.

**Fig.1.** Evolution of amplitudes of the SFH (a)$U_K(T)$, (b)$V_K(T)$ and (c) $W_K(T)$ of the AGW, when $\alpha = 0.017, K_x = K_y = 0.3, K_z = 1$.

Let us note for definiteness: according to equation (21) the normalized Brunt-Väisälä period $= 2\pi$; for the atmospheric scale heights $H = 10km$ (atmosphere below the thermosphere) and $H = 60km$ (the ionosphere F-region) single unity of the normalized time ($T$) corresponds to $\simeq 60s$ and $\simeq 150s$, respectively.

The numerical results of evolution of the shear wavelike perturbation plotted in Fig. 2 is in accordance with solution (19) of the independent evolution of these waves. In this case (see Fig. 2 and equations (19c), (19d) and (19d)) the essential changes of the amplitude of velocity SFH occur in the horizontal plane during times of $T \leq T_a$. For times of $T > T_a$ the shear wave damps. The cases demonstrated in Figs 1 and 2 are nearly same as the independent evolution of the AGW and shear waves in the horizontal shear flow.

According to solution (19) the shear waves amplitude decreases with increase of the vertical wavenumber $|K_z|$ and from equation (11) it is usually expected, that the changes of amplitudes of shear waves due to their coupling to the AGW are essential for the cases of $|K_z| > 1$.

**Fig.2.** Evolution of amplitudes of the SFH (a)$U_K(T)$, (b)$V_K(T)$ and (c) $W_K(T)$ of atmospheric shear waves in the same case as in Fig.1.

The cases of the evolution of the shear waves when the coupling to the AGW is essential are demonstrated in Figs 3 and 4. These cases clearly show that with increase of shear parameter $\alpha$ and vertical wavenumber $|K_z|$ the shear wavelike perturbation for times of $T > T_a$ transforms into the AGW. In these cases frequencies of the AGWs for great times of $T \gg T_a$ tend (analogy to the cases demonstrated in Fig. 1) to the isothermal Brunt-Väisälä frequency. In these cases with increase of $\alpha$ (Fig. 3) and $|K_z|$ (Fig. 4), comparable to the cases demonstrated in Fig. 2, the incompressible vortical perturbation located in horizontal plane – shear wave (which is essential for times of $T < T_a$ , see Figs 3 and 4 (a),(b)) transforms into the compressible perturbations – gravity waves for times of $T > T_a$ (see Figs 3 and 4 (a),(b)). For all cases demonstrated in Figs 2-4 the essential changes of horizontal velocity amplitude of shear waves are in accordance with analytical solution (19c) and (19d). This allows us to consider the importance of existence of atmospheric vortical perturbation – shear waves in the horizontal shear flow.

**Fig.3.** Evolution of amplitudes of the SFH (a)$U_K(T)$, (b)$V_K(T)$ and (c) $W_K(T)$ of atmospheric shear waves when $\alpha = 0.017, K_x = K_y = 0.3, K_z = 3$.

In the cases demonstrated in Figs 2-4 we have used values of shear parameter $\alpha$, which are in agreement with satellite mesurements (see [10]) of thermospheric winds during magnetically disturbed days. Note that in the cases when the following approximation $u_0 = u_{00} + ay$ is possible for the horizontal velocity of the winds, according to equations (2) and (8) the shear wave propagates in the meridional direction by velocity $u_{00} = const$. The last phenomenon shows the possibility of propagation of the shear wavelike vortical perturbation from polar to mid-latitudes. The appearance of shear wavelike vortical perturbation, which for times of $T \leq T_a$ gives essential changes of amplitude perturbed horizontal velocity, could be reason of the sufficient changes of height profile of the mid-latitude ionosphere F2-region electron density (see [33]). According to the cases demonstrated in Figs 2-4 the characteristic time of appearance of shear waves ($t_a$) at the ionosphere F2-region $\simeq 1 - 3h$. The observations of changes of the mid-latitude ionospheric F2-region parameters during magnetically disturbed days by time $2 - 5h$ are known (see [34]).

**Fig.4.** Evolution of amplitudes of the SFH (a)$U_K(T)$, (b)$V_K(T)$ and (c) $W_K(T)$ of atmospheric shear waves when $\alpha = 0.051, K_x = K_y = 0.3, K_z = 1$.

The analytic solutions (14)-(16) and numerical results of evolution of the AGW demonstrated in Fig. 1 and that of the shear waves demonstrated in Figs 3 and 4, show us that for large times of $T \gg T_a$ in the horizontal shear flow atmosphera accumulate energy by the Brunt-Väisälä frequency. The last phenomenon could be essential as above noted magnetically disturbed days also for the different sources of disturbances which are accompanied by the presence of the inhomogeneous horizontal wind. The accumulation of waves energy by periods of $4 - 10min$ at seismically disturbed days which was noted in Ref. [35] is possibly explained by the presence of the inhomogeneity of horizontal winds. In this case according to equations (8) and (15) for large times of $t \gg t_a$ the atmosphere accumulate energy by frequency $\omega_b - k_x u_{00}$, which values for the atmospheric scale heights $H \simeq 10km$ and for a sufficiently wide spectrum of the horizontal winds velocity $u_{00}$ could satisfy these observed results.

We can demonstrate many cases of the independent evolution of the AGWs and shear waves for the cases of $\alpha \ll 0.25$ and $|K_z| = O(1)$ when the pictures are similar to the demonstrated in Figs 1 and 2, respectively. In these cases the spectrum of the AGWs and evolution pictures of amplitude of the shear waves are in accordance with equations (14)-(16) and (19), respectively. For the cases of greater values of $|K_z| > 1$ and $\alpha < 0.25$ the coupling between gravity and shear waves gives similar pictures of evolution as demonstrated in Figs 3 and 4.

## 5. *Concluding remarks*

We have studied the linear theory of the AGW evolution in the horizontal winds with horizontal constant shear for three-dimensional case. The atmosphere is considered to be invisced, isothermally stratified and perturbation of velocity grows with height as $\rho_0^{-\frac{1}{2}}$. We have obtained characteristic dispersion equations (14)-(16) and (18) which correspond to the time varying frequencies ($\omega_g(t)$) for gravity waves and power low amplification/dampings of the amplitudes of those gravity and shear waves. The obtained AGWs frequency has minimum values at $t = \frac{t_a}{2}$ and for large times ($t \gg t_a$) tends to the its cutoff frequency – isothermal Brunt-Väisälä frequency $\omega_B$. It has been shown that the evolution of the velocity perturbation for the AGW are different in vertical and horizontal winds as well as in its shear direction. This analysis also shows that the essential amplification of the gravity wave vertical velocity amplitude occurs for times of $t > t_a$ and the dominant frequency of these waves at $t \gg t_a$ is the isothermal Brunt-Väisälä frequency. It has been shown that for atmospheric scale height $H = 60km$ and wavelengths in the meridional direction $\geq 1000km$, the obtained periods, $16 - 19min$, of the AGW evolving in the horizontal shear flow are in accordance with the observation of large scale TIDs with small periods given in Refs [26], [27] and [28].

The possibility of excitation of vortical perturbation – shear waves,located in the horizontal plane has been shown. We have also obtained the analytical expression in the case of its independent evolution in the horizontal shear flow. The time dependence of the AGW frequencies, the presence of their amplification/damping rates and appearance of the shear waves is due to the existence of the horizontal shear flow. The changes of amplitudes of the AGW and shear waves due to their coupling in the horizontal shear flow essentially depend on the shear parameter $\alpha$ and vertical wavenumbers $k_z$. The cases which correspond to the transformation of the shear waves into the AGW have been demonstrated.

It has been shown that the growth of amplitudes of the perturbed vertical velocity coressponding to the AGW are essential for times of $t > t_a$. In the horizontal shear flow the shear waves amplitude amplification is essential for times of $t \leq t_a$ and it decreases when $t > t_a$.

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