DECOMPOSITION OF SOME COMPLEX FUNCTIONS WITH RESPECT TO THE CYCLIC GROUP OF ORDER n

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Abstract

Let f be a function of the complex variable z admitting a Laurent expansion in an annulus I with center in the origin. For an arbitrary positive integer n, Ricci's theorem asserts that the function f can be written as the sum of n functions $f_{[n,k]}$, $k = 0, 1, \cdots, n-1$, defined by

$$f_{[n,k]}(z) \;=\; \frac{1}{n}\sum_{\ell=0}^{n-1} \exp\left(-\frac{2i\pi k\ell}{n}\right)\; f\left(z\exp\left(\frac{2i\pi\ell}{n}\right)\right), \quad z \in I.$$

In this paper, we shall establish certain results to derive some properties and formulas pertaining to $f_{[n,k]}$ from f ones and to express some identities of f as functions of the components $f_{[n,k]}$. As an illustration, we consider the function $f(z) = \exp(z)$. The components of this function are the hyperbolic functions of order n and k-th kind. For those functions, we obtain alternative proofs of known identities and other properties which are believed to be new.

Key words and phrases: Laurent expansion, Ricci's Theorem, hyperbolic functions.

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