ON TWO-DIMENSIONAL EQUATIONS OF THE NON-LINEAR THEORY OF SHELLS

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Abstract

In the present paper the system of two-dimensional differential equations of non-linear and non-shallow shells by means of I.N. Vekuas method is obtained.

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In his studies I.N. Vekua, by means of the method of reduction of three-dimensional problems of elasticity to two-dimensional ones, constructed several versions of the refined linear theory of thin and shallow shells, containing the regular process; thin and shallow shells were assumed to be three-dimensional elastic bodies, satisfying the following reguirements

$$a_\alpha^\beta - x_3 b_\alpha^\beta \cong a_\alpha^\beta, \; -h(x_1,x_2) \leq x_3 \leq h(x_1,x_2), \; (\alpha,\beta=1,2) \qquad (*)$$

where $a_\alpha^\beta, b_\alpha^\beta$ are mixed components of the metric tensor and tensor of curvature of the middle surface of the shell, x_3 is a thickness coordinate, h is a semithickness, depending on the curvilinear coordinates x_1, x_2 of the middle surface. Further, by expanding the unknown three-dimensional displacement and stress fields into the Fourier-Legendre series and satisfying the boundary conditions on face surfaces $x_3=\pm h$ I.N Vekua obtained the sequence of two-dimensional differential equations, containing the regular process. Besides, it is evident that every sequence will contain the unremovable error which is generated by the assumption of the form $(*)$. Therefore it is of great importance to get rid of this assumption.

In future constructions under non-shallow shells we will mean elastic bodies, free from the assumption of the form $(*)$, i.e.

$$a_\alpha^\beta - x_3 b_\alpha^\beta \neq a_\alpha^\beta, \; |x_3 b_\alpha^\beta| < 1.$$

1. Complete system of equations of the three-dimensional non-linear theory of elasticity may be written as [3, 4]

a)equilibrium equations

$$\partial_i(\sqrt{g}\boldsymbol{\sigma}^i) + \sqrt{g}\mathbf{\Phi} = 0,\ \boldsymbol{\sigma}^{\mathrm{i}} = \sigma^{\mathrm{ij}}(\mathbf{R}_{\mathrm{j}} + \partial_{\mathrm{j}}\mathbf{U}),\ (\partial_{\mathrm{i}} = \frac{\partial}{\partial \mathrm{x_i}}), \tag{1}$$

where g is a discriminant of the metric form of the space, $\boldsymbol{\sigma}^i$ are contravariant vectors of stress, $\mathbf{\Phi}$ is an external force, σ^{ij} are contravariant components of the stress tensor, $\mathbf{R}_{\mathrm{i}}$ are covariant base vectors of the space, $\mathbf{U}$ is a displacement vector.

Under repeating indexes we mean summation,the Latin letters taking the values 1,2,3 and the Greek ones-1,2.

b) Equations of the state-there exists a variety of approaches about dependence between tensors of stress and strain (Seth B.R., Signorini A., Murnaghan F.O., John F., Blatz P.J. and Ko W.L., Novozhilow B.B.,etc).

Material is called hyper-elastic, if the stress is obtained from the energy function of the strain with the relation

$$\sigma^{ij} = \frac{\partial \ni}{\partial \epsilon_{ij}},$$

where $\ni$ is an energy function of the strain, ε_{ij} are covariant components of the strain tensor.

The theory of second order hyper-elasticity has the form [5] :

$$\ni = \frac{1}{2}E^{ijpq}\varepsilon_{ij}\varepsilon_{pq} + \frac{1}{3}E^{ijpqsk}\varepsilon_{ij}\varepsilon_{pq}\varepsilon_{sk},$$

$$\varepsilon_{ij} = \frac{1}{2}(\boldsymbol{R}_i\partial_j\boldsymbol{U} + \boldsymbol{R}_j\partial_i\boldsymbol{U} + \partial_i\boldsymbol{U}\partial_j\boldsymbol{U}),$$

$$\sigma^{ij} = E^{ijpq}\varepsilon_{pq} + E^{ijpqsk}\varepsilon_{pq}\varepsilon_{sk}, \tag{2}$$

where E^{ijpq}, E^{ijpqsk} are coefficients of elastisity of the first and second orders.

For isotropic materials coefficients of elasticity of the first order are expressed only by two coefficients of Lamé

$$E^{ijpq} = \lambda g^{ij}g^{pq} + \mu(g^{ip}g^{jq} + g^{iq}g^{jp}), \tag{3}$$

and coefficients of elasticity of second order are defined by formula [5]

$$\begin{aligned} E^{ijpqsk} &= E_1 g^{ij}g^{pq}g^{sk} + E_2(g^{ij}g^{pq}g^{sk} - g^{ij}g^{pk}g^{qs}) + \\ &+ E_3 g^{ip}g^{jq}g^{sk} + E_4 g^{is}g^{pq}g^{jk}. \end{aligned} \tag{4}$$

Here E_1, E_2, E_3, and E_4 are moduli of elasticity of second order for isotropic and elastic materials.
c) Boundary conditions for the stress vector,acting on the area with the normal $\overset{*}{\mathbf{l}}$, have the form

$$\boldsymbol{\sigma}_{(\check{l})} = \boldsymbol{\sigma}^i \overset{*}{l}_i, \quad (\overset{*}{\mathbf{l}}\mathbf{R}_i = \overset{*}{l}_i). \tag{5}$$

For reducing three-dimensional problems to two-dimensional ones it is necessary to write $(1-5)$ in bases of the middle surface, therefore it is expedient to constider the coordinate system, normally connected with the middle surface,i.e.

$$\mathbf{R}(\mathrm{x}_1, \mathrm{x}_2, \mathrm{x}_3) = \mathbf{r}(\mathrm{x}_1, \mathrm{x}_2) + \mathrm{x}_3\mathbf{n}(\mathrm{x}_1, \mathrm{x}_2),$$

where $\mathbf{R}$ and $\mathbf{r}$ are radius-vectors of the space and middle surface, $\mathbf{n}$ is a normal of the surface. The dependence between covariant and contravariant base vectors of the space and the surface is exspressed as [1]:

$$\mathbf{R}_\mathrm{i} = \mathrm{A}_{\mathrm{ij}}\mathbf{r}^\mathrm{j} = \mathrm{A}_\mathrm{i}^\mathrm{j}\mathbf{r}_\mathrm{j},\ \mathbf{R}^\mathrm{i} = \mathrm{A}^{\mathrm{iJ}}\mathbf{r}_\mathrm{j} = \mathrm{A}_\mathrm{j}^\mathrm{i}\mathbf{r}^\mathrm{j},\ \mathbf{R}_3 = \mathbf{R}^3 = \mathbf{r}_3 = \mathbf{r}^3 = \mathbf{n}, \tag{6}$$

where

$$\begin{aligned}
&A_{\alpha\beta} = a_{\alpha\beta} - x_3 b_{\alpha\beta},\ \sqrt{g}A^{\alpha\beta} = \sqrt{a}\left[a^{\alpha\beta} + x_3\left(b^{\alpha\beta} - 2Ha^{\alpha\beta}\right)\right],\\
&A_{\alpha 3} = A^{\alpha 3} = 0,\ A_{33} = A^{33} = 1,\\
&A_\alpha^\beta = a_\alpha^\beta - x_3 b_\alpha^\beta,\ \sqrt{g}A_\beta^\alpha = \sqrt{a}\left[a_\beta^\alpha + x_3(b_\beta^\alpha - 2Ha_\beta^\alpha)\right],\\
&A_\alpha^3 = A_3^\alpha = 0,\ A_3^3 = 1,\\
&\sqrt{g} = \sqrt{a}(1 - 2Hx_3 + Kx_3^2),\ \sqrt{a} = a_{11}a_{22} - a_{12}^2,\ (a_{\alpha\beta} = \mathbf{r}_\alpha\mathbf{r}_\beta),\\
&(\alpha, \beta = 1, 2),
\end{aligned} \tag{7}$$

H and K are middle and Gaussian curvatures of the surface, a is a discriminant of the metric form of the surface.

Now relation (1) assumes the form

$$\frac{1}{\sqrt{a}}\frac{\partial\sqrt{a}(\sqrt{\frac{g}{a}}\boldsymbol{\sigma}^\alpha)}{\partial x_\alpha} + \frac{\partial\sqrt{\frac{g}{a}}\boldsymbol{\sigma}^3}{\partial x_3} + \sqrt{\frac{g}{a}}\boldsymbol{\Phi} = 0. \tag{8}$$

From (2),(3) and (4) we get

$$\boldsymbol{\sigma}^i = \sigma^{ij}(\mathbf{R}_\mathrm{j} + \partial_\mathrm{j}\mathbf{u}) = (\mathrm{E}^{\mathrm{ijpq}} + \mathrm{E}^{\mathrm{ijpqsk}}\varepsilon_{\mathrm{sk}})\varepsilon_{\mathrm{pq}}(\mathbf{R}_\mathrm{j} + \partial_\mathrm{j}\mathbf{u}) \Rightarrow$$

$$\Rightarrow \boldsymbol{\sigma}^i = A^i_{i_1} A^p_{p_1} \Big\{ M^{i_1 j_1 p_1 q_1} + A^k_{k_1} M^{i_1 j_1 p_1 q_1 s_1 k_1} (\boldsymbol{r}_{s_1} \partial_k \mathbf{u} + \tag{9}$$

$$+ \frac{1}{2} A^s_{s_1} \partial_s \mathbf{u} \partial_\mathrm{k} \mathbf{u}) \Big\} (\boldsymbol{r}_{q_1} \partial_p \mathbf{u} + \frac{1}{2} \mathrm{A}^\mathrm{q}_{\mathrm{q}_1} \partial_\mathrm{p} \mathbf{u} \partial_\mathrm{q} \mathbf{u})(\boldsymbol{r}_{\mathrm{j}_1} + \mathrm{A}^\mathrm{j}_{\mathrm{j}_1} \partial_\mathrm{j} \mathbf{u}),$$

where

$$M^{i_1 j_1 p_1 q_1} = \lambda a^{i_1 j_1} a^{p_1 q_1} + \mu (a^{i_1 p_1} a^{j_1 q_1} + a^{i_1 q_1} a^{j_1 p_1}),$$

$$M^{i_1 j_1 p_1 q_1 s_1 k_1} = (E_1 + E_2) a^{i_1 j_1} a^{p_1 q_1} a^{s_1 k_1} - E_2 a^{i_1 j_1} a^{p_1 s_1} a^{q_1 k_1} + \tag{10}$$

$$+ E_3 a^{i_1 p_1} a^{j_1 q_1} a^{s_1 k_1} + E_4 a^{i_1 k_1} a^{j_1 s_1} a^{p_1 q_1}.$$

The stress tensor of the lateral surface $d\hat{S} = d\hat{s}dx_3$ with the normal

$$\hat{\mathbf{l}} = [(1 - \mathrm{x}_3\mathrm{k}_\mathrm{s})\mathbf{l} - \mathrm{x}_3 \tau_\mathrm{s} \mathbf{s}] \frac{\mathrm{ds}}{\mathrm{d}\hat{\mathrm{s}}} \quad (\hat{\mathbf{s}} = \frac{\mathrm{d}\mathbf{R}}{\mathrm{d}\hat{\mathrm{s}}} = [(1 - \mathrm{x}_3\mathrm{k}_\mathrm{s})\mathbf{s} + \mathrm{x}_\mathrm{i}\tau_\mathrm{s}\mathbf{l}] \frac{\mathrm{ds}}{\mathrm{d}\hat{\mathrm{s}}})$$

has the form [10]

$$\boldsymbol{\sigma}_{(\hat{l})} = \boldsymbol{\sigma}^\alpha (\hat{\mathbf{l}} \boldsymbol{R}_\alpha) = \sqrt{\frac{\mathrm{g}}{\mathrm{a}}} \boldsymbol{\sigma}^\alpha (\boldsymbol{l} \boldsymbol{r}_\alpha) \frac{\mathrm{ds}}{\mathrm{d}\hat{\mathrm{s}}}, \quad (\hat{\mathbf{l}} \times \hat{\mathbf{s}} = \mathbf{n}), \tag{11}$$

where $\mathbf{l}$ and $\mathbf{s}$ are unit vectors of the tangential normal and tangent lateral curve of the middle surface ($\mathbf{l} \times \mathbf{s} = \mathbf{n}$), k_s and τ_s is the normal curvature and geodesic torsion of this curve, $d\hat{s}$ and ds are linear elements of the surface $x_3 = const$ and middle surface, moreover

$$d\hat{s} = \sqrt{1 - 2x_3 k_s + x_3^2 (k_s^2 + \tau_s^2)} ds.$$

The formula (11) is proved with dyadic representation of the differential $d\mathbf{R}$ of the surface $x_3 = const$ [3] :

$$d\mathbf{R} = \mathrm{d}\mathbf{r} \cdot (\nabla \otimes \mathbf{R}), \quad (\nabla = \mathbf{r}^\alpha \partial_\alpha)$$

and determination of the tangential normal $\hat{\mathbf{l}}$

$$\hat{\mathbf{l}} = \frac{\mathrm{d}\mathbf{R}}{\mathrm{d}\hat{\mathrm{s}}} \times \mathbf{n} = \frac{\mathrm{d}\mathbf{r} \cdot (\nabla \otimes \mathbf{R})}{\mathrm{ds}} \times \mathbf{n} \frac{\mathrm{ds}}{\mathrm{d}\hat{\mathrm{s}}} = \mathbf{s} \cdot (\nabla \otimes \mathbf{R}) \times \mathbf{n} \frac{\mathrm{ds}}{\mathrm{d}\hat{\mathrm{s}}} =$$

$$= s^\alpha \mathbf{R}_\alpha \times \mathbf{n} \frac{\mathrm{ds}}{\mathrm{d}\hat{\mathrm{s}}} = \mathrm{s}^\alpha \in_{\alpha 3 \beta} \mathbf{R}^\beta \frac{\mathrm{ds}}{\mathrm{ds}^\mathrm{n}} =$$

$$= s^{\alpha}\sqrt{\frac{g}{a}}\sqrt{a}\in_{\alpha 3\beta}\mathbf{R}^{\beta}\frac{\mathrm{ds}}{\mathrm{ds}^{\mathrm{n}}} = \mathrm{s}^{\alpha}\sqrt{\frac{\mathrm{g}}{\mathrm{a}}}(\mathbf{r}_{\alpha}\times\mathbf{n})\mathbf{r}_{\beta}\mathbf{R}^{\beta}\frac{\mathrm{ds}}{\mathrm{d}\hat{\mathrm{s}}} =$$

$$= \sqrt{\frac{g}{a}}(\mathbf{s}\times\mathbf{n})\mathbf{r}_{\beta}\mathbf{R}^{\beta}\frac{\mathrm{ds}}{\mathrm{d}\hat{\mathrm{s}}} = \sqrt{\frac{\mathrm{g}}{\mathrm{a}}}(\mathbf{l}\cdot\mathbf{r}_{\beta})\mathbf{R}^{\beta}\frac{\mathrm{ds}}{\mathrm{d}\hat{\mathrm{s}}} \Rightarrow$$

$$\Rightarrow \hat{\mathbf{l}}\cdot\mathbf{R}_{\beta} = \sqrt{\frac{\mathrm{g}}{\mathrm{a}}}(\mathbf{l}\cdot\mathbf{r}_{\beta})\frac{\mathrm{ds}}{\mathrm{d}\hat{\mathrm{s}}},$$

where $\in_{ijk}$ is Levi-Chivitta's symbol, $C_{ijk} = \sqrt{g}\in_{ijk}$.

2. Now, following I.N.Vekua [1], assume the validity of the following expansions

$$\left(\sqrt{\frac{g}{a}}\boldsymbol{\sigma}^{i}, \mathbf{U}, \sqrt{\frac{\mathrm{g}}{\mathrm{a}}}\bar{\mathbf{\Phi}}\right) = \sum_{m=0}^{\infty}\left(\overset{(\mathrm{m})}{\boldsymbol{\sigma}}{}^{\mathrm{i}}, \overset{(\mathrm{m})}{\mathbf{u}}, \overset{(\mathrm{m})}{\mathbf{\Phi}}\right)P_{m}\left(\frac{x_3}{h}\right),$$

where

$$\left(\overset{(\mathrm{m})}{\boldsymbol{\sigma}}{}^{\mathrm{i}}, \overset{(\mathrm{m})}{\mathbf{u}}, \overset{(\mathrm{m})}{\Phi}\right) = \frac{2m+1}{2h}\int_{-h}^{h}\left(\sqrt{\frac{g}{a}}\boldsymbol{\sigma}^{i}, \mathbf{u}, \sqrt{\frac{\mathrm{g}}{\mathrm{a}}}\mathbf{\Phi}\right)P_{m}\left(\frac{x_3}{h}\right)dx_3. \qquad (12)$$

Here P_m are Legendre's polynomials of order m.

By substituting these expansions into (8),(9),(11), satisfying beforehand the boundary conditions of the face surface $x_3 = \pm h$

$$\boldsymbol{\sigma}^{3}(x_1, x_2, \pm h) = \overset{(\pm)}{\boldsymbol{\sigma}}{}^{3},$$

and the formula

$$P'_m(x) = (2m-1)P_{m-1}(x) + (2m-5)P_{m-3}(x) + ...,$$

we get the following infinite system of two-dimensional equations:

a) equilibrium equations:

$$\nabla_{\alpha}\overset{(\mathrm{m})}{\boldsymbol{\sigma}}{}^{\alpha} - \frac{2\mathrm{m}+1}{\mathrm{h}}(\overset{(\mathrm{m}-1)}{\boldsymbol{\sigma}}{}^{3} + \overset{(\mathrm{m}-3)}{\boldsymbol{\sigma}}{}^{3} + ...) + \overset{(\mathrm{m})}{\mathbf{F}} = 0, \qquad (13)$$

where ∇_{α} are covariant derivatives on the surface, and

$$\overset{(\mathrm{m})}{\mathbf{F}} = \overset{(\mathrm{m})}{\mathbf{\Phi}} + \frac{2\mathrm{m}+1}{2\mathrm{h}}(\sqrt{\frac{\mathrm{g}_{+}}{\mathrm{a}}}\overset{(+)}{\boldsymbol{\sigma}}{}^{3} - (-1)^{\mathrm{m}}\sqrt{\frac{\mathrm{g}_{-}}{\mathrm{a}}}\overset{(-)}{\boldsymbol{\sigma}}{}^{3}),$$

$$(\sqrt{\frac{g\pm}{a}} = 1 \pm 2Hh + Kh^2);$$

b) equations of the state

$$
\overset{(m)}{\boldsymbol{\sigma}}{}^{i}=\frac{2m+1}{2h}\int\limits_{-h}^{h}\sqrt{\frac{g}{a}}\boldsymbol{\sigma}^{i}P_{m}\Big(\frac{x_3}{h}\Big)dx_3=M^{i_1j_1p_1q_1}\sum_{m_1=0}^{\infty}\Bigg\{\overset{(m)}{\underset{(m_1)}{A}}{}^{i\ p}_{i_1p_1}\times
$$

$$
\times(\mathbf{r}_{q_1}D_p\overset{(m_1)}{\mathbf{U}})\mathbf{r}_{j_1}+\sum_{m_2=0}^{\infty}\Bigg[\overset{(m)}{\underset{(m_1,m_2)}{A}}{}^{i\ j\ p}_{i_1j_1p_1}(\mathbf{r}_{q_1}D_p\overset{(m_1)}{\mathbf{U}})D_j\overset{(m_2)}{\mathbf{U}}+
$$

$$
+\frac{1}{2}\overset{(m)}{\underset{(m_1,m_2)}{A}}{}^{i\ p\ q}_{i_1p_1q_1}(D_p\overset{(m_1)}{\mathbf{U}}\cdot D_q\overset{(m_2)}{\mathbf{U}})\mathbf{r}_{j_1}+
$$

$$
+\frac{1}{2}\sum_{m_3=0}^{\infty}\overset{(m)}{\underset{(m_1,m_2,m_3)}{A}}{}^{i\ j\ p\ q}_{i_1j_1p_1q_1}(D_p\overset{(m_1)}{\mathbf{U}}\cdot D_q\overset{(m_2)}{\mathbf{U}})\cdot D_j\overset{(m_3)}{\mathbf{U}}\Bigg]\Bigg\}+
$$

$$
+M^{i_1j_1p_1q_1s_1k_1}\sum_{m_1,m_2=0}^{\infty}\Bigg\{\overset{(m)}{\underset{(m_1,m_2)}{A}}{}^{i\ p\ k}_{i_1,p_1,k_1}(\mathbf{r}_{q_1}D_p\overset{(m1)}{\mathbf{U}})(\mathbf{r}_{s_1}D_k\overset{(m_2)}{\mathbf{U}})\mathbf{r}_{j_1}+
$$

$$
+\sum_{m_3=0}^{\infty}\Bigg[\overset{(m)}{\underset{(m_1,m_2,m_3)}{A}}{}^{i\ j\ p\ k}_{i_1j_1p_1k_1}(\mathbf{r}_{q_1}D_p\overset{(m_1)}{\mathbf{U}})(\mathbf{r}_{s_1}D_k\overset{(m_2)}{\mathbf{U}})D_j\overset{(m_3)}{\mathbf{U}}+
$$

$$
+\frac{1}{2}\overset{(m)}{\underset{(m_1,m_2,m_3)}{A}}{}^{i\ p\ s\ k}_{i_1p_1s_1k_1}(\mathbf{r}_{q_1}D_p\overset{(m_1)}{\mathbf{U}})(D_s\overset{(m_2)}{\mathbf{U}}\cdot D_k\overset{(m_3)}{\mathbf{U}})\mathbf{r}_{j_1}+ \qquad (14)
$$

$$
+\frac{1}{2}\overset{(m)}{\underset{(m_1m_2m_3)}{A}}{}^{i\ p\ q\ k}_{i_1p_1q_1k_1}(D_p\overset{(m_1)}{\mathbf{U}}\cdot D_q\overset{(m_2)}{\mathbf{U}}(\mathbf{r}_{s_1}\cdot D_k\overset{(m_3)}{\mathbf{U}})\mathbf{r}_{j_1}+
$$

$$
+\frac{1}{2}\sum_{m_4=0}^{\infty}\frac{1}{2}\overset{(m)}{\underset{(m_1m_2m_3m_4)}{A}}{}^{i\ p\ q\ s\ k}_{i_1p_1q_1s_1k_1}(D_p\overset{(m_1)}{\mathbf{U}}\cdot D_q\overset{(m_2)}{\mathbf{U}})(D_s\overset{(m_3)}{\mathbf{U}}\cdot D_k\overset{(m_4)}{\mathbf{U}})\mathbf{r}_{j_1}+
$$

$$
+\overset{(m)}{\underset{(m_1,m_2,m_3,m_4)}{A}}{}^{i\ j\ p\ q\ k}_{i_1j_1p_1q_1k_1}(D_p\overset{(m_1)}{\mathbf{U}}\cdot D_q\overset{(m_2)}{\mathbf{U}})D_j\overset{(m_3)}{\mathbf{U}}(\mathbf{r}_{s_1}D_k\overset{(m_4)}{\mathbf{U}})+
$$

$$
+\overset{(m)}{\underset{(m_1m_2m_3m_4)}{A}}{}^{i\ j\ p\ k\ s}_{i_1j_1p_1k_1s_1}(\mathbf{r}_{q_1}D_p\overset{(m_1)}{\mathbf{U}})\cdot D_j\overset{(m_2)}{\mathbf{U}}\cdot(D_s\overset{(m_3)}{\mathbf{U}}\cdot D_k\overset{(m_4)}{\mathbf{U}})+
$$

$$
+\frac{1}{2}\sum_{m_5=0}^{\infty}\overset{(m)}{\underset{(m_1,m_2,m_3,m_4,m_5)}{A}}{}^{i\ j\ p\ q\ s\ k}_{i_1j_1p_1q_1s_1k_1}(D_p\overset{(m_1)}{\mathbf{U}}\cdot D_q\overset{(m_2)}{\mathbf{U}})D_j\overset{(m_3)}{\mathbf{U}}\times
$$

$$
\times(D_s\overset{(m_4)}{\mathbf{U}}D_k\overset{(m_5)}{\mathbf{U}})\Bigg]\Bigg\},
$$

where

$$D_k \overset{(\mathrm{m})}{\mathbf{U}} = \delta_{\mathrm{k}}^{\beta} \partial_{\beta} \overset{(\mathrm{m})}{\mathbf{U}} + \delta_{\mathrm{k}}^{3} \overset{(\mathrm{m})}{\mathbf{U}}{}',$$

$$\overset{(\mathrm{m})}{\mathbf{U}}{}' = \tfrac{2\mathrm{m}+1}{\mathrm{h}} (\overset{(\mathrm{m}+1)}{\mathbf{U}} + \overset{(\mathrm{m}+3)}{\mathbf{U}} + ...),$$

$$\underset{(m_1)}{\overset{(m)}{A}}{}^{i\ p}_{i_1 p_1} = \frac{2m+1}{2h} \int\limits_{-h}^{h} \sqrt{\frac{g}{a}} A^i_{i_1} A^p_{p_1} P_{m_1} P_m dx_3,$$

$$\underset{(m_1,m_2)}{\overset{(m)}{A}}{}^{i\ j\ p}_{i_1 j_1 p_1} = \frac{2m+1}{2h} \int\limits_{-h}^{h} \sqrt{\frac{g}{a}} A^i_{i_1} A^j_{j_1} A^p_{p_1} P_{m_1} P_{m_2} P_m dx_3,$$

$$\underset{(m_1,m_2,m_3)}{\overset{(m)}{A}}{}^{i\ j\ p\ q}_{i_1 j_1 p_1 q_1} = \frac{2m+1}{2h} \int\limits_{-h}^{h} \sqrt{\frac{g}{a}} A^i_{i_1} A^j_{j_1} A^p_{p_1} A^q_{q_1} P_{m_1} P_{m_2} P_{m_3} P_m dx_3,$$

$$\underset{(m_1,\dots,m_4)}{\overset{(m)}{A}}{}^{i\ j\ p\ q\ s}_{i_1 j_1 p_1 q_1 s_1} = \frac{2m+1}{2h} \int\limits_{-h}^{h} \sqrt{\frac{g}{a}} A^i_{i_1} A^j_{j_1} A^p_{p_1} A^q_{q_1} A^s_{s_1} \times \tag{15}$$

$$\times P_{m_1} P_{m_2} P_{m_3} P_{m_4} P_m dx_3,$$

$$\underset{(m_1,\dots,m_5)}{\overset{(m)}{A}}{}^{i\ j\ p\ q\ s\ k}_{i_1 j_1 p_1 q_1 s_1 k_1} = \frac{2m+1}{2h} \int\limits_{-h}^{h} \sqrt{\frac{g}{a}} A^i_{i_1} A^j_{j_1} A^p_{p_1} A^q_{q_1} A^s_{s_1} A^k_{k_1} \times$$

$$\times P_{m_1} P_{m_2} P_{m_3} P_{m_4} P_{m_5} P_m dx_3;$$

c) boundary conditions

$$\overset{(\mathrm{m})}{\boldsymbol{\sigma}}_{(l)} = \overset{(m)}{\sigma}_{(ll)} \mathbf{l} + \overset{(\mathrm{m})}{\sigma}_{(\mathrm{ls})} \mathbf{s} + \overset{(\mathrm{m})}{\sigma}_{(\mathrm{ln})} \mathbf{n} = \frac{2\mathrm{m}+1}{2\mathrm{h}} \int\limits_{-\mathrm{h}}^{\mathrm{h}} \boldsymbol{\sigma}_{(\hat{\mathrm{l}})} \frac{\mathrm{d}\hat{\mathrm{s}}}{\mathrm{ds}} \mathrm{P}_{\mathrm{m}} (\frac{\mathrm{x}_3}{\mathrm{h}}) \mathrm{dx}_3.$$

The integrals in formulae (15) are calculated in an explicit form, for example [10]

$$I_1) \underset{(m_1)}{\overset{(m)}{A}}{}^{\alpha\ \beta}_{\alpha_1 \beta_1} = \frac{2m+1}{2h} \int\limits_{-h}^{h} \sqrt{\frac{g}{a}} A^{\alpha}_{\alpha_1} A^{\beta}_{\beta_1} P_{m_1} (\frac{x_3}{h}) P_m (\frac{x_3}{h}) dx_3 =$$

$$= \frac{(b^{\alpha}_{\alpha_1} - 2Ha^{\alpha}_{\alpha_1})(b^{\beta}_{\beta_1} - 2Ha^{\beta}_{\beta_1})}{K} \delta^m_{m_1} + \frac{2m+1}{2h\sqrt{E}} \times$$

$$\times\left[B^{\alpha}_{\alpha_1}(y)B^{\beta}_{\beta_1}(y)\begin{pmatrix} P_{m_1}(y)Q_m(y), m_1 \le m \\ P_m(y)Q_{m_1}(y), m_1 \ge m \end{pmatrix}\right]^{y_2}_{y_1},$$

where Q_m are Legendre's functions of the second order, when

$$\begin{gathered} B^{\alpha}_{\alpha_1}(y) = a^{\alpha}_{\alpha_1} + hy(b^{\alpha}_{\alpha_1} - 2Ha^{\alpha}_{\alpha_1}),\ \ y_{1,2} = ((H \mp \sqrt{E})h)^{-1}, \\ E = H^2 - K,\ [f(y)]^{y_2}_{y_1} = f(y_2) - f(y_1),\ \ (\alpha, \beta, \alpha_1, \beta_1 = 1, 2). \end{gathered} \tag{16}$$

Here we have used the formulae of F.Neumann and J.Adams [6]

$$\int\limits_{-1}^{1} \frac{P_m(t)dt}{x - t} = 2Q_m(x),\ (|x| > 1),$$

$$P_m(x)P_n(x) = \sum_{r=0}^{\min(m,n)} \alpha_{mnr}P_{m+n-2r}(x),$$

$$\left(\alpha_{mnr} = \frac{A_{m-r}A_rA_{n-r}}{A_{m+n-r}} \cdot \frac{2(m+n) - 4r + 1}{2(m+n) - 2r + 1},\right.$$
$$\left. A_m = \frac{1.3...2m-1}{m!}\right),$$

as well as the representations

$$\frac{1}{1 - 2Hx_3 + Kx_3^2} = \begin{cases} \dfrac{1}{2\sqrt{E}}(\dfrac{1}{\frac{1}{H+\sqrt{E}} - x_3} - \dfrac{1}{\frac{1}{H-\sqrt{E}} - x_3}), \\ \qquad E = H^2 - K \neq 0; \\ \dfrac{1}{(1 - Hx_3)^2},\ E = H^2 - K = 0. \end{cases}$$

Further,

$$I_2)\ \overset{(m)}{\underset{(m_1)}{A}}{}^{\alpha\ ,3}_{\alpha_1,3} = \frac{2m+1}{2h}\int\limits_{-h}^{h}\sqrt{\frac{g}{a}}A^{\alpha}_{\alpha_1}P_{m_1}(\frac{x_3}{h})P_m(\frac{x_3}{h})dx_3 =$$

$$= \frac{2m+1}{2h}\int\limits_{-h}^{h}[a^{\alpha}_{\alpha_1} + x_3(b^{\alpha}_{\alpha_1} - 2Ha^{\alpha}_{\alpha_1})]P_{m_1}P_m dx_3 =$$

$$= a^{\alpha}_{\alpha_1}\delta^m_{m_1} + (b^{\alpha}_{\alpha_1} - 2Ha^{\alpha}_{\alpha_1})h(\frac{m}{2m-1}\delta^{m-1}_{m_1} + \frac{m+1}{2m+3}\delta^{m+1}_{m_1});$$

$$I_3)\ \overset{(m)}{\underset{(m_1)}{A}}{}^{33}_{33} = \frac{2m+1}{2h}\int\limits_{-h}^{h}\sqrt{\frac{g}{a}}P_{m_1}(\frac{x_3}{h})P_m(\frac{x_3}{h})dx_3 =$$

$$=\frac{2m+1}{2h}\int\limits_{-h}^{h}(1-2Hx_3+Kx_3^2)P_{m_1}(\frac{x_3}{h})P_m(\frac{x_3}{h})dx_3=$$

$$=\delta_{m_1}^m-2Hh(\frac{m}{2m-1}\delta_{m_1}^{m-1}+\frac{m+1}{2m+3}\delta_{m_1}^{m+1})+Kh^2[\frac{(m+1)(m+2)}{(2m+3)(2m+5)}\delta_{m_1}^{m+2}+$$

$$+\frac{2m^2+2m-1}{(2m+3)(2m-1)}\delta_{m_1}^m+\frac{m(m-1)}{(2m-1)(2m-3)}\delta_{m_1}^{m-2}].$$

Let us write out the integrals, containing the product of three Legendre's polynomials:

$$II_1)\ \underset{(m_1,m_2)}{\overset{(m)}{A}}{}_{\alpha_1\beta_1\gamma_1}^{\alpha\ \beta\ \gamma}=\frac{2m+1}{2h}\int\limits_{-h}^{h}\sqrt{\frac{g}{a}}A_{\alpha_1}^{\alpha}A_{\beta_1}^{\beta}A_{\gamma_1}^{\gamma}P_{m_1}P_{m_2}P_m dx_3=$$

$$=\frac{2m+1}{2h}\int\limits_{-h}^{h}\frac{B_{\alpha_1}^{\alpha}(\frac{x_3}{h})B_{\beta_1}^{\beta}(\frac{x_3}{h})B_{\gamma_1}^{\gamma}(\frac{x_3}{h})}{(1-2Hx_3+kx_3^2)^2}P_{m_1}(\frac{x_3}{h})P_{m_2}(\frac{x_3}{h})P_m(\frac{x_3}{h})dx_3=$$

$$=\frac{2m+1}{h}\frac{1}{k^2h^4}\sum_{r=0}^{m_1}\alpha_{m_1m_2r}\frac{\partial^2}{\partial y_1\partial y_2}\frac{1}{r_1-r_2}\times$$

$$\times\left[B_{\alpha_1}^{\alpha}(y)B_{\beta_1}^{\beta}(y)B_{\gamma_1}^{\gamma}(y)\left\{\begin{array}{l}P_{m_1+m_2-2r}(y)Q_m(y),m_1+m_2-2r\le m\\ Q_{m_1+m_2-2r}(y)P_m(y),m_1+m_2-2r\ge m\end{array}\right\}\right]_{y_1}^{y_2},$$

where $B_{\beta}^{\alpha}(\frac{x_3}{h})=a_{\beta}^{\alpha}+x_3(b_{\beta}^{\alpha}-2Ha_{\beta}^{\alpha})$

$$II_2)\ \underset{(m_1m_2)}{\overset{(m)}{A}}{}_{\alpha_1\beta_1 3}^{\alpha\ \beta\ 3}=\frac{2m+1}{2h}\int\limits_{-h}^{h}\sqrt{\frac{g}{a}}A_{\alpha_1}^{\alpha}A_{\beta_1}^{\beta}P_{m_1}P_{m_2}P_m dx_3=$$

$$=\sum_{r=0}^{min(m_1,m_2)}\alpha_{m_1m_2r}\left\{\frac{(b_{\alpha_1}^{\alpha}-2Ha_{\alpha_1}^{\alpha})(b_{\beta_1}^{\beta}-2Ha_{\beta_1}^{\beta})}{K}\delta_{m_1+m_2-2r}^m+\frac{2m+1}{2h}\frac{1}{\sqrt{E}}\times\right.$$

$$\left.\times\left[B_{\alpha_1}^{\alpha}(y)B_{\beta_1}^{\beta}(y)\left(\begin{array}{l}P_{m_1+m_2+2r}(y)Q_m(y),m_1+m_2-2r\le m\\ P_m(y)Q_{m_1+m_2-2r}(y),m_1+m_2-2r\ge m\end{array}\right)\right]_{y_1}^{y_2}\right\},$$

$$II_3)\ \underset{(m_1,m_2)}{\overset{(m)}{A}}{}_{\alpha_1 33}^{\alpha\ 33}=\frac{2m+1}{2h}\int\limits_{-h}^{h}\sqrt{\frac{g}{a}}A_{\alpha_1}^{j}P_{m_1}P_{m_2}P_m dx_3=$$

$$= \sum_{r=0}^{min(m_1,m_2)} \alpha_{m_1 m_2 r} \left\{ a^{\alpha}_{\alpha_1} \delta^{m}_{m_1+m_2-2r} + h \left(a^{\alpha}_{\alpha_1} - 2H a^{\alpha}_{\alpha_1} \right) \left(\frac{m+1}{2m+3} \delta^{m+1}_{m_1+m_2-2r} + \right.\right.$$

$$\left.\left. + \frac{m}{2m-1} \delta^{m-1}_{m_1+m_2-2r} \right) \right\};$$

$$II_4) \quad \underset{(m_1,m_2)}{\overset{(m)}{A}} {}^{333}_{333} = \frac{2m+1}{2h} \int_{-h}^{h} \sqrt{\frac{g}{a}} P_{m_1} P_{m_2} P_m dx_3 =$$

$$= \sum_{r=0}^{min(m_1,m_2)} \alpha_{m_1 m_2 r} \left\{ \delta^{m}_{m_1+m_2-2r} - 2Hh \left(\frac{m+1}{2m+3} \delta^{m+1}_{m_1+m_2-2r} + \right.\right.$$

$$\left. + \frac{m}{2m-1} \delta^{m-1}_{m_1+m_2-2r} \right) + K h^2 \left[\frac{(m+1)(m+2)}{(2m+3)(2m+5)} \delta^{m+2}_{m_1+m_2-2r} + \right.$$

$$\left.\left. + \frac{2m^2+2m-1)}{(2m+3)(2m-1)} \delta^{m}_{m_1+m_2-2r} + \frac{m(m-1)}{(2m-1)(2m-3)} \delta^{m-2}_{m_1+m_2-2r} \right] \right\}.$$

In particular, if $m_1 + m_2 \leq m$, we will have

$$\underset{(m_1,m_2)}{\overset{(m)}{A}} {}^{\alpha\ ,\beta\ ,\gamma}_{\alpha_1,\beta_1,\gamma_1} = \frac{2m+1}{k^2 h^4} \frac{\partial^2}{\partial y_1 \partial y_2} \left\{ \frac{1}{y_1 y_2} \left[B^{\alpha}_{\alpha_1}(y) B^{\beta}_{\beta_1}(y) B^{\gamma}_{\gamma_1}(y) B^{\delta}_{\delta_1}(y) \times \right.\right.$$

$$\left.\left. \times P_{m_1}(y) P_{m_2}(y) Q_m(y) \right]^{y_2}_{y_1} \right\}.$$

Let us write out the integrals, containing the product of four Legendre's polynomials:

$$III_1) \quad \underset{(m_1,m_2,m_3)}{\overset{(m)}{A}} {}^{\alpha\ \beta\ \gamma\ \delta}_{\alpha_1 \beta_1 \gamma_1 \delta_1} = \frac{2m+1}{2h} \int_{-h}^{h} \sqrt{\frac{g}{a}} A^{\alpha}_{\alpha_1} A^{\beta}_{\beta_1} A^{\gamma}_{\gamma_1} A^{\delta}_{\delta_1} P_{m_1}(\frac{x_3}{h}) \times$$

$$\times P_{m_2}(\frac{x_3}{h}) P_{m_3}(\frac{x_3}{h}) P_m(\frac{x_3}{h}) dx_3 =$$

$$= \frac{2m+1}{2 \cdot 4} \frac{1}{k^3 h^6} \sum_{r_1=0}^{min(m_1,m_2)} \sum_{r_2=0}^{min(m,m_3)} \alpha_{m_1 m_2 r_1} \alpha_{m m_3 r_2} \frac{\partial^4}{\partial y_1^2 \partial y_2^2} \left(\frac{1}{y_1 \cdot y_2} \times \right.$$

$$\times \left[B^{\alpha}_{\alpha_1}(y) B^{\beta}_{\beta_1}(y) B^{\gamma}_{\gamma_1}(y) B^{\delta}_{\delta_1}(y) \times \right.$$

$$\left.\left. \times \left\{ \begin{array}{l} P_{m_1+m_2-2r_1}(y) Q_{m+m_3-2r_2}(y), m_1+m_2-2r_1 \leq m+m_3-2r_2 \\ Q_{m_1+m_2-2r_1}(y) P_{m+m_3-2r_2}(y), m_1+m_2-2r_1 \geq m+m_3-2r_2 \end{array} \right\} \right]^{y_2}_{y_1} \right);$$

$$III_2)\quad \overset{(m)}{\underset{(m_1,m_2,m_3)}{A}}{}^{\alpha\ \beta\ \gamma\ 3}_{\alpha_1\beta_1\gamma_1 3} = \frac{2m+1}{2h}\int\limits_{-h}^{h}\sqrt{\frac{g}{a}}A^{\alpha}_{\alpha_1}A^{\beta}_{\beta_1}A^{\gamma}_{\gamma_1}P_{m_1}(\frac{x_3}{h})P_{m_2}(\frac{x_3}{h})\times$$

$$\times P_{m_3}(\frac{x_3}{h})P_m(\frac{x_3}{h})dx_3 =$$

$$= \frac{2m+1}{k^2h^4}\sum_{r_1=0}^{min(m_1,m_2)}\alpha_{m_1m_2r_1}\sum_{r_2=0}^{min(m,m_3)}\alpha_{mm_3r_2}\frac{\partial^2}{\partial y_1\partial y_2}\left(\frac{1}{y_1\cdot y_2}\times\right.$$

$$\times\left[B^{\alpha}_{\alpha_1}(y)B^{\beta}_{\beta_1}(y)B^{\gamma}_{\gamma_1}(y)\times\right.$$

$$\left.\left.\times\left\{\begin{array}{l} P_{m_1+m_2-2r_1}(y)Q_{m+m_3-2r_2}(y), m_1+m_2-2r_1\leq m+m_3-2r_2 \\ Q_{m_1+m_2-2r_1}(y)P_{m+m_3-2r_2}(y), m_1+m_2-2r_1\geq m+m_3-2r_2\end{array}\right\}\right]_{y_1}^{y_2}\right);$$

$$III_3)\quad \overset{(m)}{\underset{(m_1,m_2,m_3)}{A}}{}^{\alpha\ \beta\ 33}_{\alpha_1\beta_1 33} = \frac{2m+1}{2h}\int\limits_{-h}^{h}\sqrt{\frac{g}{a}}A^{\alpha}_{\alpha_1}A^{\beta}_{\beta_1}P_{m_1}(\frac{x_3}{h})P_{m_2}(\frac{x_3}{h})\times$$

$$\times P_{m_3}(\frac{x_3}{h})P_m(\frac{x_3}{h})dx_3 =$$

$$= \frac{2m+1}{2}\sum_{r_1=0}^{min(m_1,m_2)}\alpha_{m_1m_2r_1}\sum_{r_2=0}^{min(m,m_3)}\alpha_{mm_3r_2}\left\{\frac{1}{\sqrt{E}}\left[B^{\alpha}_{\alpha_1}(y)B^{\beta}_{\beta_1}(y)\times\right.\right.$$

$$\left.\left.\times\left\{\begin{array}{l} P_{m_1+m_2-2r_1}(y)Q_{m+m_3-2r_2}(y), m_1+m_2-2r_1\leq m+m_3-2r_2 \\ Q_{m_1+m_2-2r_1}(y)P_{m+m_3-2r_2}(y), m_1+m_2-2r_1\geq m+m_3-2r_2\end{array}\right\}\right]_{y_1}^{y_2}\right\};$$

$$III_4)\quad \overset{(m)}{\underset{(m_1,m_2,m_3)}{A}}{}^{\alpha\ 333}_{\alpha_1 333} = \frac{2m+1}{2h}\int\limits_{-h}^{h}\sqrt{\frac{g}{a}}A^{\alpha}_{\alpha_1}P_{m_1}(\frac{x_3}{h})P_{m_2}(\frac{x_3}{h})P_{m_3}(\frac{x_3}{h})\times$$

$$\times P_m(\frac{x_3}{h})dx_3 = \sum_{r_1=0}^{min(m_1,m_2)}\alpha_{m_1m_2r_1}\sum_{r_2=0}^{min(m,m_3)}\alpha_{mm_3r_2}\times$$

$$\times\left\{a^{\alpha}_{\alpha_1}\frac{2m+1}{2(m+m_3-2r_2)+1}\delta^{m+m_3-2r_2}_{m_1+m_2-2r_1}+h\left(b^{\alpha}_{\alpha_1}-2Ha^{\alpha}_{\alpha_1}\right)\times\right.$$

$$\times\left[\frac{m+m_3-2r_2+1}{2(m+m_3-2r_2)+1}\cdot\frac{2m+1}{2(m+m_3-2r_2)+3}\delta^{m+m_3-2r_2+1}_{m_1+m_2-2r_1}+\right.$$

$$\left.\left.+\frac{m+m_3-2r_2}{2(m+m_3-2r_2)+1}\cdot\frac{2m+1}{2(m+m_3-2r_2)+1}\delta^{m+m_3-2r_2+1}_{m_1+m_2-2r_1}\right]\right\};$$

$$III_5)\ \overset{(m)}{\underset{(m_1,m_2,m_3)}{A}}{}^{3333}_{3333}=\frac{2m+1}{2h}\int\limits_{-h}^{h}(1-2Hx_3+Kx_3)P_{m_1}(\frac{x_3}{h})\times$$

$$\times P_{m_2}(\frac{x_3}{h})P_{m_3}(\frac{x_3}{h})P_m(\frac{x_3}{h})dx_3=$$

$$=\sum_{r_1=0}^{min(m_1,m_2)}\alpha_{m_1m_2r_1}\sum_{r_2=0}^{min(m,m_3)}\alpha_{mm_3r_2}\left\{\frac{2m+1}{2(m+m_3-2r_2)+1}\delta^{m+m_3-2r_2}_{m_1+m_2-2r_1}-\right.$$

$$-2Hh\left[\frac{m+m_3-2r_2+1}{2(m+m_3-2r_2)+1}\cdot\frac{2m+1}{2(m+m_3-2r_2)+3}\delta^{m+m_3-r_2}_{m_1+m_2-2r_1}+\right.$$

$$\left.+\frac{m+m_3-2r_2}{2(m+m_3-2r_2)+1}\cdot\frac{2m+1}{2(m+m_3-2r_2)-1}\delta^{m+m_3-2r_2-1}_{m_1+m_2-2r_1}\right]+$$

$$+Kh^2\left[\frac{m+m_3-2r_2+1}{2(m+m_3-2r_2)+1}\cdot\frac{m+m_3-2r_2+2}{2(m+m_3-2r_2)+3}\times\right.$$

$$\times\frac{2m+1}{2(m+m_3-2r_2)+5}\delta^{m+m_3-2r_2+2}_{m_1+m_2-2r_1}+$$

$$+\left(\frac{(m+m_3-2r_2+1)^2}{(2(m+m_3-2r_2)+1)(2(m+m_3-2r_2)+3)}+\right.$$

$$\left.+\frac{(m+m_3-2r_2)^2}{(2(m+m_3-2r_2)+1)(2(m+m_3-2r_2)-1)}\right)\times$$

$$\times\frac{2m+1}{2(m+m_3-2r_2)+1}\delta^{m+m_3-r_2}_{m_1+m_2-2r_1}+$$

$$+\frac{m+m_3-2r_2}{2(m+m_3-2r_2)+1}\cdot\frac{m+m_3-2r_2+1}{2(m+m_3-2r_2)-1}\times$$

$$\left.\left.\times\frac{2m+1}{2(m+m_3-2r_2)-3}\delta^{m+m_3-2r_2-2}_{m_1+m_2-2r_1}\right]\right\}.$$

Then, let us consider the integrals,containing the product of five Legendre's polynomials:

$$IV_1)\ \overset{(m)}{\underset{(m_1,\ldots,m_4)}{A}}{}^{\alpha\ \beta\ \gamma\ \delta\ \eta}_{\alpha_1\beta_1\gamma_1\delta_1\eta_1}=\frac{2m+1}{2h}\int\limits_{-h}^{h}\sqrt{\frac{g}{a}}A^{\alpha}_{\alpha_1}A^{\beta}_{\beta_1}A^{\gamma}_{\gamma_1}A^{\delta}_{\delta_1}A^{\eta}_{\eta_1}P_{m_1}(\frac{x_3}{h})\times$$

$$\times P_{m_2}(\frac{x_3}{h})P_{m_3}(\frac{x_3}{h})P_{m_4}(\frac{x_3}{h})P_m(\frac{x_3}{h})dx_3=\frac{2m+1}{3!^2}\frac{1}{k^4h^8}\sum_{r_1=0}^{min(m_1,m_2)}\alpha_{m_1m_2r_1}\times$$

$$\times \sum_{r_2=0}^{min(m_3,m_4)} \alpha_{m_3 m_4 r_2} \sum_{r_3=0}^{m_1+m_2-2r_1} \alpha_{m_1+m_2-2r_1, m_3+m_4-2r_2, r_3} \times$$

$$\times \frac{\partial^6}{\partial y_1^3 \partial y_2^3} \frac{1}{y_1-y_2} \Big[B^{\alpha}_{\alpha_1}(y) B^{\beta}_{\beta_1}(y) B^{\gamma}_{\gamma_1}(y) B^{\delta}_{\delta_1}(y) B^{\eta}_{\eta_1}(y) \times$$

$$\times \left\{ \begin{array}{l} P_{m_1+...+m_4-2(r_1+r_2+r_3)}(y) Q_m(y), m_1+...+m_4-2(r_1+r_2+r_3) \le m \\ Q_{m_1+...+m_4-2(r_1+r_2+r_3)}(y) P_m(y), m_1+...+m_4-2(r_1+r_2+r_3) \ge m \end{array} \right\} \Bigg]_{y_1}^{y_2};$$

$$IV_2) \underset{(m_1,...,m_4)}{\overset{(m)}{A}} {}^{\alpha\ \beta\ \gamma\ \delta\ 3}_{\alpha_1 \beta_1 \gamma_1 \delta_1 3} = \frac{2m+1}{2h} \int_{-h}^{h} \sqrt{\frac{g}{a}} A^{\alpha}_{\alpha_1} A^{\beta}_{\beta_1} A^{\gamma}_{\gamma_1} A^{\delta}_{\delta_1} P_{m_1} P_{m_2} P_{m_3} P_{m_4} P_m dx_3 =$$

$$= \frac{2m+1}{4} \cdot \frac{1}{k^3 h^6} \frac{\partial^4}{\partial y_1^2 \partial y_2^2} \frac{1}{y_1-y_2} \sum_{r_1=0}^{min(m_1,m_2)} \alpha_{m_1 m_2 r_1} \sum_{(r_2=0)}^{min(m_3,m_4)} \alpha_{m_3 m_4 r_2} \times$$

$$\times \sum_{r_3=0}^{m_1+m_2-2r_1} \alpha_{m_1+m_2-2r_1, m_3+m_4-2r_2, r_3} [B^{\alpha}_{\alpha_1}(y) B^{\beta}_{\beta_1}(y) B^{\gamma}_{\gamma_1}(y) B^{\delta}_{\delta_1}(y) \times$$

$$\times \left\{ \begin{array}{l} P_{m_1+...+m_4-2(r_1+r_2+r_3)}(y) Q_m(y), m_1+...+m_4-2(r_1+r_2+r_3) \le m \\ Q_{m_1+...+m_4-2(r_1+r_2+r_3)}(y) P_m(y), m_1+...+m_4-2(r_1+r_2+r_3) \ge m \end{array} \right\} \Bigg]_{y_1}^{y_2};$$

$$IV_3) \underset{(m_1,...,m_4)}{\overset{(m)}{A}} {}^{\alpha\ \beta\ \gamma\ 33}_{\alpha_1 \beta_1 \gamma_1 33} = \frac{2m+1}{2h} \int_{-h}^{h} \sqrt{\frac{g}{a}} A^{\alpha}_{\alpha_1} A^{\beta}_{\beta_1} A\gamma_{\gamma_1} P_{m_1} P_{m_2} P_{m_3} P_{m_4} P_m dx_3 =$$

$$= \frac{2m+1}{k^2 h^4} \sum_{r_1=0}^{min(m_1 m_2)} \alpha_{m_1 m_2 r_1} \sum_{r_2=0}^{min(m_3 m_4)} \alpha_{m_3 m_4 r_2} \sum_{r_3=0}^{m_1+m_2-2r_1} \alpha_{m_1+m_2-2r_1, m_3+m_4-2r_2, r_3} \times$$

$$\times \frac{\partial^2}{\partial y_1 \partial y_2} \frac{1}{y_1-y_2} \Big[B^{\alpha}_{\alpha_1}(y) B^{\beta}_{\beta_1}(y) B^{\gamma}_{\gamma_1}(y) \times$$

$$\times \left\{ \begin{array}{l} P_{m_1+...+m_4-2(r_1+r_2+r_3)}(y) Q_m(y), m_1+...+m_4-2(r_1+r_2+r_3) \le m, \\ Q_{m_1+...+m_4-2(r_1+r_2+r_3)}(y) P_m(y), m_1+...+m_4-2(r_1+r_2+r_3) \ge m \end{array} \right\} \Bigg]_{y_1}^{y_2};$$

$$IV_4) \underset{(m_1,...,m_4)}{\overset{(m)}{A}} {}^{\alpha\ \beta\ 333}_{\alpha_1 \beta_1 333} = \frac{2m+1}{2h} \int_{-h}^{h} \sqrt{\frac{g}{a}} A^{\alpha}_{\alpha_1} A^{\beta}_{\beta_1} P_{m_1} P_{m_2} P_{m_3} P_{m_4} P_m dx_3 =$$

$$= \sum_{r_1=0}^{min(m_1 m_2)} \alpha_{m_1 m_2 r_1} \sum_{r_2=0}^{min(m_3 m_4)} \alpha_{m_3 m_4 r_2} \sum_{r_3=0}^{min(m_1+m_2-2r_1)} \alpha_{m_1+m_2-2r_1, m_3+m_4-2r_2, r_3} \times$$

$$\times\left\{\frac{2m+1}{2\sqrt{E}h}\left[B^{\alpha}_{\alpha_1}(y)B^{\beta}_{\beta_1}(y)\times\right.\right.$$

$$\left.\times\left\{\begin{array}{ll} P_{m_1+...+m_4-2(r_1+r_2+r_3)}(y)Q_m(y), m_1+...+m_4-2(r_1+r_2+r_3)\le m \\ Q_{m_1+...+m_4-2(r_1+r_2+r_3)}(y)P_m(y), m_1+...+m_4-2(r_1+r_2+r_3)\ge m \end{array}\right\}\right]_{y_1}^{y_2}+$$

$$\left.+\frac{(b^{\alpha}_{\alpha_1}-2Ha^{\alpha}_{\alpha_1})(b^{\beta}_{\beta_1}-2Ha^{\beta}_{\beta_1})}{K}\delta^m_{m_1+...+m_4-2(r_1+r_2+r_3)}\right\};$$

$$IV_5)\ \underset{(m_1,...,m_4)}{\overset{(m)}{A}}{}^{\alpha\ 3333}_{\alpha_1 3333}=\frac{2m+1}{2h}\int\limits_{-h}^{h}\sqrt{\frac{g}{a}}A^{\alpha}_{\alpha_1}P_{m_1}P_{m_2}P_{m_3}P_{m_4}P_m dx_3=$$

$$=\sum_{r_1=0}^{min(m_1,m_2)}\alpha_{m_1m_2r_1}\sum_{r_2=0}^{min(m_3,m_4)}\alpha_{m_3m_4r_2}\sum_{r_3=0}^{m_1+m_2-2r_1}\alpha_{m_1+m_2-2r_1,m_3+m_4-2r_2,r_3}\times$$

$$\left\{a^{\alpha}_{\alpha_1}\delta^m_{m_1+...+m_4-2(r_1+r_2+r_3)}++h(b^{\alpha}_{\alpha_1}-2Ha^{\alpha}_{\alpha_1})\times\right.$$

$$\left.\times\left[\frac{m+1}{2m+3}\delta^{m+1}_{m_1+...+m_4-2(r_1+r_2+r_3)}+\frac{m}{2m+1}\delta^{m-1}_{m_1+...+m_4-2(r_1+r_2+r_3)}\right]\right\};$$

$$IV_6)\ \underset{(m_1,...,m_4)}{\overset{(m)}{A}}{}^{33333}_{33333}=\frac{2m+1}{2h}\int\limits_{-h}^{h}(1-2Hx_3+Kx_3)^2P_{m_1}P_{m_2}P_{m_3}P_{m_4}P_m dx_3=$$

$$=\sum_{r_1=0}^{min(m_1,m_2)}\alpha_{m_1m_2r_1}\sum_{r_2=0}^{min(m_3,m_4)}\alpha_{m_3m_4r_2}\sum_{r_3=0}^{m_1+m_2-2r_1)}\alpha_{m_1+m_2-2r_1,m_3+m_4-2r_2,r_3}\times$$

$$\times\left\{\delta^m_{m_1+...+m_4-3(r_1+r_2+r_3)}-2Hh\left[\frac{m+1}{3m+3}\delta^{m-1}_{m_1+...+m_4-2(r_1+r_2+r_3)}+\right.\right.$$

$$\left.+\frac{m}{2m-1}\delta^{m-1}_{m_1+...+m_4-2(r_1+r_2+r_3)}\right]+$$

$$+Kh^2\left[\frac{(m+1)(m+2)}{(2m+3)(2m+5)}\delta^{m+2}_{m_1+...+m_4-2(r_1+r_2+r_3)}+\right.$$

$$\left.\left.+\frac{2m^2+2m-1}{(2m+3)(2m-4)}\delta^m_{m_1+...}+\frac{m(m-1)}{(2m-1)(2m-3)}\delta^{m-2}_{m_1+...}\right]\right\}.$$

For the integrals,containing the product of six Legendre's polynomials, we have

$$V_1)\ \underset{(m_1...m_5)}{\overset{(m)}{A}}{}^{\alpha_1\alpha_2\alpha_3\alpha_4\alpha_5\alpha_6}_{\beta_1\beta_2\beta_3\beta_4\beta_5\beta_6}=\frac{2m+1}{2h}\int_{-h}^{h}\sqrt{\frac{g}{a}}A^{\alpha_1}_{\beta_1}...A^{\alpha_6}_{\beta_6}P_{m_1}(\frac{x_3}{h})...P_{m_5}(\frac{x_3}{h})\times$$

$$\times P_m(\frac{x_3}{h})dx_3 == \frac{2m+1}{(4!)^2}\frac{1}{k^6h^{10}}\frac{\partial^8}{\partial y_1^4\partial y_2^4}\frac{1}{y_1-y_2}\sum_{r_1=0}^{m_1}\alpha_{m_1m_2r_1}\sum_{r_2=0}^{m_3}\alpha_{m_3m_4r_2}\times$$

$$\times\sum_{r_3=0}^{m_1+m_2-2r_1}\alpha_{m_1+m_2-2r_1,m_3+m_4-2r_2,r_3}\sum_{r_4=0}^{m_5}\alpha_{m_1+...+m_4-2(r_1+...+r_3),m_5,r_4}\times$$

$$\times\Big[B_{\beta_1}^{\alpha_1}(y)...B_{\beta_6}^{\alpha_6}(y)\times$$

$$\times\left\{\begin{array}{l} P_{m_1+...+m_5-2(r_1+...+r_4)}(y)Q_m(y), m_1+...+m_5-2(r_1+...+r_4)\le m \\ Q_{m_1+...+m_5-2(r_1+...+r_4)}(y)P_m(y), m_1+...+m_5-2(r_1+...+r_4)\ge m\end{array}\right\}\Bigg]_{y_1}^{y_2};$$

$$V_2)\ \overset{(m)}{A}(m_1...m_5)\,{}^{\alpha_1\alpha_2\alpha_3\alpha_4\alpha_5 3}_{\beta_1\beta_2\beta_3\beta_4\beta_5\ 3} = \frac{2m+1}{2h}\int_{-h}^{h}\sqrt{\frac{g}{a}}A_{\beta_1}^{\alpha_1}A_{\beta_2}^{\alpha_2}A_{\beta_3}^{\alpha_3}A_{\beta_4}^{\alpha_4}A_{\beta_5}^{\alpha_5}P_{m_1}(\frac{x_3}{h})\times$$

$$\times P_{m_2}(\frac{x_3}{h})P_{m_3}(\frac{x_3}{h})P_{m_4}(\frac{x_3}{h})P_{m_5}(\frac{x_3}{h})P_m(\frac{x_3}{h})dx_3 =$$

$$= \underset{(m_1,...m_5)}{\overset{(m)}{A}}\,{}^{\alpha_1...\alpha_5 3}_{\beta_1...\beta_5 3} = \frac{2m+1}{(3!)^2}\cdot\frac{1}{k^4h^8}\cdot\frac{\partial^6}{\partial y_1^3\partial y_2^3}\cdot\frac{1}{y_1-y_2}\sum_{r_1=0}^{m_1}\alpha_{m_1m_2r_1}\sum_{r_2=0}^{m_3}\alpha_{m_3m_4r_2}\times$$

$$\times\sum_{r_3=0}^{m_1+m_2-2r_1}\alpha_{m_1+m_2-2r_1,m_3+m_4-2r_2,r_3}\sum_{r_4=0}^{m_5}\alpha_{m_1+..+m_4-2(r_1+r_2+r_3),m_5,r_4}\times$$

$$\times\Big[B_{\beta_1}^{\alpha_1}(y)...B_{\beta_5}^{\alpha_5}(y)\times$$

$$\times\left\{\begin{array}{l} P_{m_1+...+m_5-2(r_1+...+r_4)}(y)Q_m(y), m_1+...+m_5-2(r_1+...+r_4)\le m \\ Q_{m_1+...+m_5-2(r_1+...+r_4)}(y)P_m(y), m_1+...+m_5-2(r_1+...+r_4)\ge m\end{array}\right.\Bigg]_{y_1}^{y_2};$$

$$V_3)\ \underset{(m_1,...m_5)}{\overset{(m)}{A}}\,{}^{\alpha_1\alpha_2\alpha_3\alpha_4 33}_{\beta_1\beta_2\beta_3\beta_4\ 33} = \frac{2m+1}{2h}\int_{-h}^{h}\sqrt{\frac{g}{a}}A_{\beta_1}^{\alpha_1}A_{\beta_2}^{\alpha_2}A_{\beta_3}^{\alpha_3}A_{\beta_4}^{\alpha_4}P_{m_1}(\frac{x_3}{h})...P_{m_5}\times$$

$$\times P_m(\frac{x_3}{h})dx_3 = \frac{2m+1}{4}\cdot\frac{1}{k^3h^6}\cdot\frac{\partial^4}{\partial y_1^2\partial y_2^2}\cdot\frac{1}{y_1-y_2}\sum_{r_1=0}^{m_1}\alpha_{m_1m_2r_1}\sum_{r_2=0}^{m_3}\alpha_{m_3m_4r_2}\times$$

$$\times\sum_{r_3=0}^{m_1+m_2-2r_1}\alpha_{m_1+m_2-2r_1,m_3+m_4-2r_2,r_3}\sum_{r_4=0}^{m_5}\alpha_{m_1+...+m_4-2(r_1+r_2+r_3),m_5,m}\times$$

$$\times\Big[B_{\beta_1}^{\alpha_1}(y)...B_{\beta_4}^{\alpha_4}(y)\times$$

$$\times\left\{\begin{array}{l} P_{m_1+...+m_5-2(r_1+...+r_4)}(y)Q_m(y), m_1+...+m_5-2(r_1+...+r_4)\le m \\ Q_{m_1+...+m_5-2(r_1+...+r_4)}(y)P_m(y), m_1+...+m_5-2(r_1+...+r_4)\ge m\end{array}\right\}\Bigg]_{y_1}^{y_2};$$

$$V_4)\ \overset{(m)}{\underset{(m_1...m_5)}{A}}{}^{\alpha_1\alpha_2\alpha_3 333}_{\beta_1\beta_2\beta_3\ 333} = \frac{2m+1}{2h}\int\limits_{-h}^{h}\sqrt{\frac{g}{a}}A^{\alpha_1}_{\beta_1}A^{\alpha_2}_{\beta_2}A^{\alpha_3}_{\beta_3}P_{m_1}(\frac{x_3}{h})...P_{m_5}(\frac{x_3}{h})\times$$

$$\times P_m(\frac{x_3}{h})dx_3 = \frac{2m+1}{k^2h^4}\cdot\frac{\partial^2}{\partial y_1\partial y_2}\cdot\frac{1}{y_1-y_2}\sum_{r_1=0}^{m_1}\alpha_{m_1m_2r_1}\sum_{r_2=0}^{m_3}\alpha_{m_3m_4r_2}\times$$

$$\times\sum_{r_3=0}^{m_1+m_2-2r_1}\alpha_{m_1+m_2-2r_1,m_3+m_4-2r_2,r_3}\sum_{r_4=0}^{N_5}\alpha_{m_1+...+m_4-2(r_1+r_2+r_3),m_5,r_4}\times$$

$$\times\Big[B^{\alpha_1}_{\beta_1}(y)B^{\alpha_2}_{\beta_2}(y)B^{\alpha_3}_{\beta_3}(y)\times$$

$$\times\left\{\begin{array}{l} P_{m_1+...+m_5-2(r_1+...+r_4)}(y)Q_m(y), m_1+...+m_5-2(r_1+...+r_4)\le m \\ Q_{m_1+...+m_5-2(r_1+...+r_4)}(y)P_m(y), m_1+...+m_5-2(r_1+...+r_4)\ge m\end{array}\right.\Bigg]_{y_1}^{y_2};$$

$$V_5)\ \overset{(m)}{\underset{(m_1...m_5)}{A}}{}^{\alpha_1\alpha_2 333}_{\beta_1\beta_2 333} = \frac{2m+1}{2h}\int\sqrt{\frac{g}{a}}A^{\alpha_1}_{\beta_1}A^{\alpha_2}_{\beta_2}P_{m_1}(\frac{x_3}{h})...P_{m_5}(\frac{x_3}{h})P_m(\frac{x_3}{h})dx_3 =$$

$$= \sum_{r_1=0}^{m_1}\alpha_{m_1m_2r_1}\sum_{r_2=0}^{m_3}\alpha_{m_3m_4r_2}\sum_{r_3=0}^{m_1+m_2-2r_1}\alpha_{m_1+m_2-2r_1,m_3+m_4-2r_2,r_3}\times$$

$$\times\sum_{r_4=0}^{m_5}\alpha_{m_1+...+m_4-2(r_1+r_2+r_3),m_5,r_4}\Bigg\{\frac{2m+1}{2\sqrt{E}h}\times$$

$$\times\Big[B^{\alpha_1}_{\beta_1}(y)B^{\alpha_2}_{\beta_2}(y)\times$$

$$\times\left(\begin{array}{l} P_{m_1+...m_5-2(r_1+...r_4)}(y)Q_m(y),\ m_1+...m_5-2(r_1+...r_4)\le m \\ Q_{m_1+...m_5-2(r_1+...r_4)}(y)P_m(y),\ m_1+...m_5-2(r_1+...r_4)\ge m\end{array}\right)\Bigg]_{y_1}^{y_2}+$$

$$+\frac{(b^{\alpha}_{\alpha_1}-2Ha^{\alpha}_{\alpha_1)})(b^{\alpha_2}_{\beta_2}-2Ha^{\alpha_2}_{\beta_2})}{K}\delta^m_{m_1+...m_5-2(r_1+...r_4)}\Bigg\};$$

$$V_6)\ \overset{(m)}{\underset{(m_1,,m_5)}{A}}{}^{\alpha_1 33333}_{\beta_1 33333} = \frac{2m+1}{2h}\int\limits_{-h}^{h}\sqrt{\frac{g}{a}}A^{\alpha}_{\alpha_1}P_{m_1}...P_{m_5}P_m dx_3 =$$

$$= \sum_{r_1=0}^{m_1}\alpha_{m_1m_2r_1}\sum_{r_2=0}^{m_3}\alpha_{m_3m_4r_2}\sum_{r_3=0}^{m_1+m_2-2r_1}\alpha_{m_1+m_2-2r_1,m_3+m_4-2r_2,r_3}\times$$

$$\times\sum_{r_4=0}^{m_5}\alpha_{m_1+...m_4-2(r_1+r_2+r_3),m_5,r_4}[a^{\alpha}_{\alpha_1}\delta^{m}_{m_1+...m_5-2(r_1+...r_4)}+$$

$$+h(b^{\alpha}_{\alpha_1}-2Ha^{\alpha}_{\alpha_1})(\frac{m+1}{2m+3}\delta^{m+1}_{m_1+...+m_5-2(r_1+...r_4)}+\frac{m}{2m-1}\delta^{m-1}_{m_1+...m_5-2(r_1+...+r_4)})];$$

$$V_7)\ \underset{(m_1...m_5)}{\overset{(m)}{A}}{}^{333333}_{333333}=\frac{2m+1}{2h}\int_{-h}^{h}\sqrt{\frac{g}{a}}P_{m_1}(\frac{x_3}{h})...P_{m_5}(\frac{x_3}{h})P_m(\frac{x_3}{h})dx_3=$$

$$=\frac{2m+1}{2}\int_{-1}^{1}(1-2Hht+Kh^2t^2)\sum_{r_1=0}^{m_1}\alpha_{m_1m_2r_1}\times$$

$$\times\sum_{r_2=0}^{m_3}\alpha_{m_3m_4r_2}P_{m_1+m_2-2r_1}(t)P_{m_3+m_4-2r_2}(t)P_{m_5}(t)P_m(t)dt=$$

$$=\sum_{r_1=0}^{(m_1)}\alpha_{m_1m_2r_1}\sum_{r_2=0}^{(m_1)}\alpha_{m_3m_4r_2}\sum_{r_3=0}^{(m_1+m_2-2r_1)}\alpha_{m_1+m_2-2r_1,m_3+m_4-2r_2,r_3}\times$$

$$\times\sum_{r_4=0}^{m_5}\alpha_{m_1+...+m_4-2(r_1+r_2+r_3),m_5,r_4}\left\{\delta^{m}_{m_1+...m_5-2(r_1+...r_4)}-\right.$$

$$-2Hh\left(\frac{m+1}{2m+3}\delta^{m+1}_{m_1+...+m_5-2(r_1+...+r_4)}+\frac{m}{2m-1}\delta^{m-1}_{m_1+...+m_5-2(r_1+...r_4)}\right)+Kh^2\times$$

$$\times\left[\frac{(m+1)(m+2)}{(2m+3)(2m+5)}\delta^{m+2}_{m_1+...+m_5-2(r_1+...+r_4)}+\right.$$

$$+\frac{2m^2+2m-1}{(2m+3)(2m+1)}\delta^{m}_{m_1+...+m_5-2(r_1+...+r_4)}+$$

$$\left.\left.+\frac{m(m+1)}{(2m-1)(2m+3)}\delta^{m-2}_{m_1+...+m_5+2(r_1+...+r_4)}\right]\right\}.$$

The transform to a finite system is carried out by considering finite segments in the expansion (12), where $m=0,1,...N$.

In conclusion, the finite system of two-dimensional equations of physical and geometrical non-linear and isotropic non-shallow shells has the form:
a) equilibrium equations of the components of stress tensor (13)

$$\nabla_\alpha\overset{(m)}{\sigma}{}^{\alpha\beta}-b^{\beta}_{\alpha}\overset{(m)}{\sigma}{}^{\alpha3}-\frac{2m+1}{h}(\overset{(m-1)}{\sigma}{}^{3\beta}+\overset{(m-3)}{\sigma}{}^{3\beta}+...)+\overset{(m)}{F}{}^{\beta}=0,$$

$$\nabla_\alpha\overset{(m)}{\sigma}{}^{\alpha_3}+b_{\alpha\beta}\overset{(m)}{\sigma}{}^{\alpha\beta}-\frac{2m+1}{h}(\overset{(m-1)}{\sigma}{}^{33}+\overset{(m-3)}{\sigma}{}^{33}+...)+\overset{(m)}{F}{}^{3}=0.$$

$$(m=0,1,...N);$$

b) equations of the state

$$\overset{(m)}{\sigma}{}^{in} = M^{i_1j_1p_1q_1} \sum_{m_1=0}^{N} \Big\{ \overset{(m)}{\underset{(m_1)}{A}}{}^{i\ p}_{i_1p_1} (\mathbf{r}_{q_1} \mathrm{D_p} \overset{(m_1)}{\mathbf{u}}) \mathrm{a}^{\mathrm{n}}_{\mathrm{j}_1} +$$

$$+ \sum_{m_2=0}^{N} \Big[\overset{(m)}{\underset{(m_1,m_2)}{A}}{}^{i\ j\ p}_{i_1j_1p_1} (\mathbf{r}_{q_1} \mathrm{D_p} \overset{(m_1)}{\mathbf{u}})(\mathbf{r}^{\mathrm{n}} \mathrm{D_j} \overset{m_1}{\mathbf{u}}) +$$

$$+ \frac{1}{2} \overset{(m)}{\underset{(m_1,m_2)}{A}}{}^{i\ p\ q}_{i_1p_1q_1} (D_p \overset{(m_1)}{\mathbf{u}} \cdot \mathrm{D_q} \overset{(m_2)}{\mathbf{u}}) \mathrm{a}^{\mathrm{n}}_{\mathrm{j}_1} +$$

$$+ \frac{1}{2} \sum_{m_3=0}^{N} \overset{(m)}{\underset{(m_1m_2m_3)}{A}}{}^{i\ j\ p\ q}_{i_1j_1p_1q_1} (D_p \overset{(m_1)}{\mathbf{u}} \cdot \mathrm{D_q} \overset{(m_2)}{\mathbf{u}})(\mathbf{r}^{\mathrm{n}} \mathrm{D_j} \overset{(m_3)}{\mathbf{u}} \Big\} +$$

$$+ M^{i_1j_1p_1q_1s_1k_1} \sum_{m_2,m_2=0}^{N} \Big\{ \overset{(m)}{\underset{(m_1m_2)}{A}}{}^{i\ p\ k}_{i_1p_1k_1} (\mathbf{r}_{q_1} \mathrm{D_p} \overset{(m_1)}{\mathbf{u}})(\mathbf{r}_{s_1} \mathrm{D_k} \overset{(m_2)}{\mathbf{u}}) \mathrm{a}^{\mathrm{n}}_{\mathrm{j}_1} +$$

$$+ \sum_{m_3=0}^{N} \Big[\overset{(m)}{\underset{(m_1,m_2,m_3)}{A}}{}^{i\ \ j\ p\ k}_{i_1j_1p_1k_1} (\mathbf{r}_{q_1} \mathrm{D_p} \overset{(m_1)}{\mathbf{u}})(\mathbf{r}_{s_1} \mathrm{D_k} \overset{(m_2)}{\mathbf{u}})(\mathbf{r}^{\mathrm{n}} \mathrm{D_j} \overset{(m_3)}{\mathbf{u}}) +$$

$$+ \frac{1}{2} \overset{(m)}{\underset{(m_1m_2m_3)}{A}}{}^{i\ p\ s\ k}_{i_1p_1s_1k_1} (\mathbf{r}_{i_1} \mathrm{D_p} \overset{(m_1)}{\mathbf{u}})(\mathrm{D_s} \overset{(m_2)}{\mathbf{u}} \cdot \mathrm{D_k} \overset{(m_3)}{\mathbf{u}}) \mathrm{a}^{\mathrm{n}}_{\mathrm{j}_1} +$$

$$+ \frac{1}{2} \overset{(m)}{\underset{(m_1,m_2,m_3)}{A}}{}^{i\ p\ q\ k}_{i_1p_1q_1k_1} (D_p \overset{(m_1)}{\mathbf{u}} \cdot \mathrm{D_q} \overset{(m_2)}{\mathbf{u}})(\mathbf{r}_{\mathrm{s}} \mathrm{D_k} \overset{(m_3)}{\mathbf{u}}) \mathrm{a}^{\mathrm{n}}_{\mathrm{j}_1} +$$

$$+ \frac{1}{2} \sum_{m_4=0}^{N} \frac{1}{2} \overset{(m)}{\underset{(m_1,m_2,m_3,m_4)}{A}}{}^{i\ p\ q\ s\ k}_{i_1p_1q_1s_1k_1} (D_p \overset{(m_1)}{\mathbf{u}} \cdot \mathrm{D_q} \overset{(m_2)}{\mathbf{u}})(\mathrm{D_s} \overset{(m_3)}{\mathbf{u}} \cdot \mathrm{D_k} \overset{(m_4)}{\mathbf{u}}) \mathrm{a}^{\mathrm{n}}_{\mathrm{j}_1} +$$

$$+ \overset{(m)}{\underset{(m_1,..,m_4,)}{A}}{}^{i\ j\ p\ q\ k}_{i_1j_1p_1q_1k_1} (D_p \overset{(m_1)}{\mathbf{u}} \cdot \mathrm{D_q} \overset{(m_2)}{\mathbf{u}})(\mathbf{r}^{\mathrm{n}} \mathrm{D_j} \overset{(m_3)}{\mathbf{u}})(\mathbf{r}_{\mathrm{s}}, \mathrm{D_k} \overset{(m_4)}{\mathbf{u}}) +$$

$$+ \overset{(m)}{\underset{(m_1,...,m_4)}{A}}{}^{i\ j\ p\ k\ s}_{i_1j_1p_1k_1s_1} (\mathbf{r}_{q_1} \mathrm{D_p} \overset{(m_1)}{\mathbf{u}})(\mathbf{r}^{\mathrm{n}} \mathrm{D_j} \overset{(m_2)}{\mathbf{u}})(\mathrm{D_s} \overset{(m_3)}{\mathbf{u}} \cdot \mathrm{D_k} \overset{(m_4)}{\mathbf{u}}) +$$

$$+ \frac{1}{2} \sum_{m_5=0}^{N} \overset{(m)}{\underset{(m_1,...,m_5)}{A}}{}^{i\ j\ p\ q\ s\ k}_{i_1j_1p_1q_1s_1k_1} (D_p \overset{(m_1)}{\mathbf{u}} \cdot \mathrm{D_q} \overset{(m_2)}{\mathbf{u}})(\mathbf{r}^{\mathrm{n}} \mathrm{D_j} \overset{(m_3)}{\mathbf{u}})(\mathrm{D_s} \overset{(m_4)}{\mathbf{u}} \cdot \mathrm{D_k} \overset{(m_5)}{\mathbf{u}}) \Big] \Big\};$$

c) boundary conditions on the contours Γ

$$\overset{(m)}{\sigma}_{(ll)} = \overset{(m)}{f_1}, \ \overset{(m)}{\sigma}_{(ls)} = \overset{(m)}{f_2}, \ \overset{(m)}{\sigma}_{(ln)} = \overset{(m)}{f_3}$$

$$(m = 0, 1, ...N).$$

3. Let's consider shallow shells, i.e.

$$a_\alpha^\beta - x_3 b_\alpha^\beta \cong a_\alpha^\beta, R_\alpha \cong r_\alpha, R^\alpha \cong r^\alpha, \sqrt{g} \cong \sqrt{a},$$

then the integrals (15) are simplified and the system of aquations of non-linear shell theory takes the form

a) equilibrium equations have the same form

b) equations of the state

$$\overset{(m)}{\sigma}{}^{in} = M^{ijpq}\{(\mathbf{r}_\mathrm{q}\mathrm{D_p}\overset{(\mathrm{m})}{\mathbf{u}})\delta_\mathrm{j}^\mathrm{n} + \sum_{\mathrm{m_1 m_2=0}}^{\mathrm{N}}[(\mathbf{r}_\mathrm{q}\mathrm{D_p}\overset{(\mathrm{m_1})}{\mathbf{u}})(\mathbf{r}^\mathrm{n}\mathrm{D_j}\overset{\mathrm{m_2}}{\mathbf{u}})+$$

$$+\frac{1}{2}(D_p\overset{\mathrm{m_1}}{\mathbf{u}}\cdot\mathrm{D_q}\overset{\mathrm{m_2}}{\mathbf{u}})\mathrm{a_j^n}]\sum_{\mathrm{r=0}}^{\mathrm{min(m_1,m_2)}}\alpha_{\mathrm{m_1 m_2 r}}\delta^\mathrm{m}_{\mathrm{m_1+m_2-2r}}+$$

$$+\frac{1}{2}\sum_{m_1,m_2,m_3=0}^{N}(D_p\overset{(\mathrm{m_1})}{\mathbf{u}}\cdot\mathrm{D_q}\overset{(\mathrm{m_2})}{\mathbf{u}})\mathrm{D_j}\overset{(\mathrm{m_2})}{\mathbf{u}}\sum_{\mathrm{r_1=0}}^{\mathrm{min(m_1,m_2)}}\alpha_{\mathrm{m_1 m_2 r_1}}\times$$

$$\times\sum_{r_2=0}^{m_3}\alpha_{m_1+m_2-2r_1,m_3,r_2}\delta^m_{m_1+m_2+m_3-2(r_1+r_2)}\}+$$

$$+M^{ijpqsk}\left\{\sum_{m_1,m_2=0}^{N}(\mathbf{r}_\mathrm{q}\mathrm{D_p}\overset{(\mathrm{m_1})}{\mathbf{u}})(\mathbf{r}_\mathrm{s}\mathrm{D_k}\overset{(\mathrm{m_2})}{\mathbf{u}})\mathrm{a_j^n}\sum_{\mathrm{r_1=0}}^{\mathrm{min(m_1,m_2)}}\alpha_{\mathrm{m_1 m_2 r}}\delta^\mathrm{m}_{\mathrm{m_1+m_2-2r}}+\right.$$

$$+\sum_{m_1,m_2,m_3=0}^{N}\Big[(\mathbf{r}_\mathrm{q}\mathrm{D_p}\overset{\mathrm{m_1}}{\mathbf{u}})\mathrm{D_j}(\mathbf{r}^\mathrm{n}\mathrm{D_j}\overset{(\mathrm{m_2})}{\mathbf{u}})(\mathbf{r}_\mathrm{s}\mathrm{D_k}\overset{(\mathrm{m_2})}{\mathbf{u}}) + \frac{1}{2}(\mathbf{r}_\mathrm{q}\mathrm{D_p}\overset{(\mathrm{m_1})}{\mathbf{u}})(\mathrm{D_s}\overset{(\mathrm{m_2})}{\mathbf{u}}\times$$

$$\times D_k\overset{(\mathrm{m_3})}{\mathbf{u}})\mathrm{a_j^n} + \frac{1}{2}(\mathrm{D_p}\overset{(\mathrm{m_1})}{\mathbf{u}}\cdot\mathrm{D_q}\overset{(\mathrm{m_2})}{\mathbf{u}})(\mathbf{r}_\mathrm{s}\cdot\mathrm{D_k}\overset{(\mathrm{m_2})}{\mathbf{u}})\mathrm{a_j^n}]\sum_{\mathrm{r_1=0}}^{\mathrm{min(m_1,m_2)}}\alpha_{\mathrm{m_1 m_2 r_1}}\times$$

$$\times\sum_{r_2=0}^{min(m_1,m_2)}\alpha_{m_1+m_2-2r_1,m_3,r_1}\delta^m_{m_1+m_2+m_3-2(r_1+r_2)}+$$

$$+\sum_{m_1,...,m_4=0}^{N}[\frac{1}{4}(D_p\overset{(\mathrm{m_1})}{\mathbf{u}}\cdot\mathrm{D_q}\overset{(\mathrm{m_2})}{\mathbf{u}})(\mathrm{D_s}\overset{(\mathrm{m_3})}{\mathbf{u}}\cdot\mathrm{D_k}\overset{(\mathrm{m_4})}{\mathbf{u}})\mathrm{a_j^n}+$$

$$+\frac{1}{2}(\mathbf{r}_\mathrm{q}\mathrm{D_p}\overset{(\mathrm{m_1})}{\mathbf{u}})(\mathbf{R}^\mathrm{n}\mathrm{D_j}\overset{(\mathrm{m_2})}{\mathbf{u}})(\mathrm{D_s}\overset{(\mathrm{m_3})}{\mathbf{u}}\cdot\mathrm{D_k}\overset{(\mathrm{m_4})}{\mathbf{u}})+$$

$$+\frac{1}{2}(D_p\overset{(\mathrm{m_1})}{\mathbf{u}}\cdot\mathrm{D_q}\overset{(\mathrm{m_2})}{\mathbf{u}})(\mathbf{r}_\mathrm{j}\mathrm{D_k}\overset{(\mathrm{m_3})}{\mathbf{u}})(\mathbf{r}_\mathrm{s}\mathrm{D_k}\overset{(\mathrm{m_4})}{\mathbf{u}})]\sum_{\mathrm{r_1=0}}^{\mathrm{min(m_1 m_2)}}\alpha_{\mathrm{m_1 m_2 r_1}}\times$$

$$\times \sum_{r_2=0}^{min(m_3,m_4)} \alpha_{m_3 m_4 r_2} \sum_{r_3=0}^{min(m_3 m_4)} \alpha_{m_1+m_2-2r_1, m_3+m_4-2r_2, r_3} \delta^m_{m_1+\ldots+m_4-2(r_1+r_2+r_3)} +$$

$$+\frac{1}{4} \sum_{(m_1 \ldots m_5)=0}^{N} (D_p \overset{(\mathrm{m}_1)}{\mathbf{u}} \cdot \mathrm{D_q} \overset{(\mathrm{m}_2)}{\mathbf{u}})(\mathbf{r}_\mathrm{n} \mathrm{D_j} \overset{(\mathrm{m}_3)}{\mathbf{u}})(\mathrm{D_s} \overset{(\mathrm{m}_4)}{\mathbf{u}} \cdot \mathrm{D_k} \overset{(\mathrm{m}_5)}{\mathbf{u}}) \sum_{\mathrm{r}_1=0}^{\min(\mathrm{m}_1 \mathrm{m}_2)} \alpha_{\mathrm{m}_1 \mathrm{m}_2 \mathrm{r}_1} \times$$

$$\times \sum_{r_2=0}^{min(m_3,m_4)} \alpha_{m_3 m_4 r_2} \sum_{r_3=0}^{m_1+m_2-2r_1} \alpha_{m_1+m_2-2r_1, m_3+m_4-2r_2, r_3} \times$$

$$\times \sum_{r_4=0}^{min(m_3 m_4)} \alpha_{m_1+\ldots+m_4-2(r_1+r_2+r_3), m_5, r_4} \delta^m_{m_1+\ldots+m_5-2(r_1+\ldots+r_4)} \}.$$

c) boundary conditions on the contours Γ have the same form.
These equations are more simplified for the plates:
a) equilibrium equations take the form

$$\partial_\alpha \overset{(\mathrm{m})}{\boldsymbol{\sigma}}{}^{\alpha} - \tfrac{2\mathrm{m}+1}{\mathrm{h}} (\overset{(\mathrm{m}-1)}{\boldsymbol{\sigma}}{}^{3} + \overset{(\mathrm{m}-3)}{\boldsymbol{\sigma}}{}^{3} + \ldots) + \overset{(\mathrm{m})}{\mathbf{F}} = 0$$
$$(m = 0, 1, \ldots N);$$

b) equations of the state have the same form as the shallow shells, only the formulae for M^{ijpq} *and* M^{ijpqsk} take the form

$$M^{ijpq} = \lambda \delta^{ij} \delta^{pq} + \mu(\delta^{ip} \delta^{jq} + \delta^{iq} \delta^{ip}),$$

$$M^{ijpqsk} = E_1 \delta^{ij} \delta^{pq} \delta^{sk} + E_2(\delta^{ij} \delta^{pq} \delta^{sk} - \delta^{ij} \delta^{pk} \delta^{qs}) + E_3 \delta^{ip} \delta^{jq} \delta^{sk} + E_4 \delta^{is} \delta^{pq} \delta^{jk}.$$

4. Now, we obtain the non-shallow spherical shells in the linear case. Consider the isometric system of coordinates on the spherical surface

$$ds^2 = \Lambda(dx_1^2 + dx_2^2),$$

where

$$\Lambda = \frac{4R^2}{(1 + x_1^2 + x_2^2)^2}.$$

Introducing the complex differential operators of the form

$$2\partial_z = \partial_1 - i\partial_2, \ 2\partial_{\bar{z}} = \partial_1 + i\partial_2, \ (z = x_1 + ix_2, \ \partial_\alpha = \frac{\partial}{\partial x_\alpha}),$$

the equilibrium equations (13) and Hooke's law for a spherical shell take the following form ($\overset{(m)}{F} = 0$) :

$$\frac{1}{\Lambda} \partial_z \left(\overset{(m)}{\sigma}_{11} - \overset{(m)}{\sigma}_{22} + 2i \overset{(m)}{\sigma}_{12} \right) + \partial_{\bar{z}} \overset{(m)}{\sigma}{}^{\alpha}_{\alpha} + \frac{1}{R} \overset{(m)}{{}_{(+)}\sigma} -$$

$$-\frac{2m+1}{h}\left(\overset{(m-1)}{\sigma}{}_{+}+\overset{(m-3)}{\sigma}{}_{+}+\cdots\right)=0, \tag{17}$$

$$\frac{1}{\Lambda}\left(\partial_z\,{}_{(+)}\overset{(m)}{\sigma}+\partial_{\bar z}\,{}_{(+)}\overset{(m)}{\sigma}\right)-\frac{1}{R}\overset{(m)}{\sigma}{}_{\alpha}^{\alpha}-\frac{2m+1}{h}\left(\overset{(m-1)}{\sigma}{}_{3}^{3}+\overset{(m-3)}{\sigma}{}_{3}^{3}+\cdots\right)=0,$$

where

$$\overset{(m)}{\sigma}_{11}-\overset{(m)}{\sigma}_{22}+2i\,\overset{(m)}{\sigma}_{12}=4\mu\lambda\partial_{\bar z}\overset{(m)}{U}{}^{+},$$

$$\overset{(m)}{\sigma}{}_{\alpha}^{\alpha}=2(\lambda+\mu)\left(\overset{(m)}{\theta}+\frac{2}{R}\overset{(m)}{U}_3\right)+2\Lambda\left(\overset{(m)}{U}{}'_{3}+\frac{1}{R}\overset{(m)}{U}{}''_{3}\right),$$

$${}_{(+)}\overset{(m)}{\sigma}=\mu\left(2\partial_{\bar z}\overset{(m)}{U}_3-\frac{1}{R}\overset{(m)}{U}_{+}+\overset{(m)}{U}{}'_{+}+\frac{1}{R}\overset{(m)}{U}{}''_{+}\right),$$

$$\overset{(m)}{\sigma}_{+}=\mu\left\{2\partial_{\bar z}\overset{(m)}{U}_3-\frac{1}{R}\overset{(m)}{U}_{+}+\frac{h}{R}\left[\frac{m}{2m-1}\left(2\partial_{\bar z}\overset{(m-1)}{U}{}_{3}-\frac{1}{R}\overset{(m-1)}{U}{}_{+}\right)+\frac{m+1}{2m+3}\times\right.\right.$$

$$\left.\left.\times\left(2\partial_{\bar z}\overset{(m+1)}{U}-\frac{1}{R}\overset{(m+1)}{U}{}_{+}\right)\right]+\overset{(m)}{U}{}'_{+}+\frac{2}{R}\overset{(m)}{U}{}''_{+}+\frac{1}{R^2}\overset{(m)}{U}{}'''_{+}\right\},$$

$$\overset{(m)}{\sigma}{}_{3}^{3}=\lambda\left\{\overset{(m)}{\theta}+\frac{2}{R}\overset{(m)}{U}_3+\frac{h}{R}\left[\frac{m}{2m-1}\left(\overset{(m-1)}{\theta}+\frac{2}{R}\overset{(m-1)}{U}{}_{3}\right)+\frac{m+1}{2m+3}\times\right.\right.$$

$$\left.\left.\times\left(\overset{(m-1)}{\theta}+\frac{2}{R}\overset{(m+1)}{U}{}_{3}\right)\right]\right\}+(\lambda+2\mu)\left(\overset{(m)}{U}{}'_{3}+\frac{2}{R}\overset{(m)}{U}{}''_{3}+\frac{1}{R^2}\overset{(m)}{U}{}'''_{3}\right),$$

$$(m=0,1,2,...). \tag{18}$$

Here the following notation is used:

$$\overset{(m)}{\sigma}_{+}=\overset{(m)}{\sigma}_{31}+i\,\overset{(m)}{\sigma}_{32}=\overset{(m)}{\sigma}_{13}+i\,\overset{(m)}{\sigma}_{23},$$

$$\overset{(m)}{U}{}^{+}=\overset{(m)}{U}{}^{1}+i\,\overset{(m)}{U}{}^{2},\ \ \overset{(m)}{U}_{+}=\overset{(m)}{U}_1+i\,\overset{(m)}{U}_2,$$

$$\overset{(m)}{\theta}=\frac{1}{\Lambda}\left(\partial_z\overset{(m)}{U}_{+}+\partial_{\bar z}\overset{(m)}{U}_{+}\right),\ \ \overset{(\mathrm{m})}{\mathbf{U}}=\overset{(\mathrm{m})}{\mathrm{U}}{}^{\alpha}\mathbf{r}_{\alpha}+\overset{(\mathrm{m})}{\mathrm{U}}{}^{3}\mathbf{n},\ \ \overset{(\mathrm{m})}{\boldsymbol{\sigma}}{}^{\mathrm{i}}=\overset{(\mathrm{m})}{\sigma}{}^{\mathrm{i}\alpha}\mathbf{r}_{\alpha}+\overset{(\mathrm{m})}{\sigma}{}^{\mathrm{i3}}\mathbf{n}.$$

Thus an infinite system of two-dimensional equations is obtained for non-shallow spherical shells. There are many different ways of passing from an infinite system (17),(18) to finite systems.

Following I. Vekua, we assume

$$\left(\mathbf{u}, \sqrt{\frac{g}{a}}\boldsymbol{\sigma}^i\right) \cong \sum_{m=0}^{2N+1} \left(\overset{(\mathrm{m})}{\mathbf{U}}, \overset{(\mathrm{m})}{\boldsymbol{\sigma}}{}^{\mathrm{i}}\right) P_m\left(\frac{x_3}{h}\right), \tag{19}$$

where N is a fixed non-negative integer. The system (17) and (18),where $m = 0, 1, ..., 2N+1$ will be called an approximation of order $2N+1$.

In papers [7] and [8] the general representations were found for finite systems of equations of shallow spherical shells. From [8] it was possible to find general representations for non-shallow spherical shells.

The system (17) and (18) will be written out for even and odd values of the index m:

$$\begin{gathered}\frac{1}{\Lambda}\partial_z\left(\overset{(2k)}{\sigma}_{11} - \overset{(2k)}{\sigma}_{22} + 2i\overset{(2k)}{\sigma}_{12}\right) + \partial_{\bar z}\overset{(2k)}{\sigma}{}_{\alpha}^{\alpha} + \\ +\frac{1}{R}\,{}_{(+)}\overset{(2k)}{\sigma} - \frac{4k+1}{h}\sum_{n=0}^{k-1}\overset{(2n+1)}{\sigma}{}_{+} = 0,\end{gathered} \tag{20}$$

$$\frac{1}{\Lambda}\left(\partial_{z(+)}\overset{(2k)}{\sigma} + \partial_{\bar z(+)}\overset{(2k)}{\sigma}\right) - \frac{1}{R}\overset{(2k)}{\sigma}{}_{\alpha}^{\alpha} - \frac{4k+1}{h}\sum_{n=0}^{k-1}\overset{(2n+1)}{\sigma}{}_{3}^{3} = 0,$$

$$\begin{gathered}\frac{1}{\Lambda}\partial_z\left(\overset{(2k+1)}{\sigma}{}_{11} - \overset{(2k+1)}{\sigma}{}_{22} + 2i\overset{(2k+1)}{\sigma}{}_{12}\right) + \\ +\partial_{\bar z}\overset{(2k+1)}{\sigma}{}_{\alpha}^{\alpha} + \frac{1}{R}\,{}_{(+)}\overset{(2k+1)}{\sigma} - \frac{4k+3}{h}\sum_{n=0}^{k}\overset{(2n)}{\sigma}{}_{+} = 0,\end{gathered} \tag{21}$$

$$\frac{1}{\Lambda}\left(\partial_z\,{}_{(+)}\overset{(2k+1)}{\sigma} + \partial_{\bar z}\,{}_{(+)}\overset{(2k+1)}{\sigma}\right) - \frac{1}{R}\overset{(2k+1)}{\sigma}{}_{\alpha}^{\alpha} - \frac{4k+1}{h}\sum_{n=0}^{k}\overset{(2n)}{\sigma}{}_{3}^{3} = 0,$$

$$(k = 0, 1, ...N, \ \ \Lambda = 4R^2/(1+z\bar z)^2),$$

where

$$\overset{(2k)}{\sigma}_{11} - \overset{(2k)}{\sigma}_{22} + 2i\overset{(2k)}{\sigma}_{12} = 4\mu\Lambda\partial_{\bar z}\overset{(2k)}{U}{}_{+}, \ \ \overset{(2k)}{\sigma}{}_{\alpha}^{\alpha} = 2(\lambda+\mu)\left(\overset{(2k)}{\theta} + \frac{2}{R}\overset{(2k)}{U}_3\right) +$$

$$+2\Lambda\left[-\frac{2k+1}{R}\overset{(2k)}{U}_3 + (4k+1)\sum_{n=k}^{N}\left(\frac{1}{h}\overset{(2n+1)}{U}_3 + \frac{1}{R}\overset{(2n)}{U}_3\right)\right],$$

$${}_{(+)}\overset{(2k)}{\sigma} = \mu\left\{2\partial_{\bar z}\overset{(2k)}{U}_3 - 2\frac{k+1}{R}\overset{(2k)}{U}_{+} + \right.$$

$$+(4k+1)\sum_{n=k}^{N}\left(\frac{1}{h}\overset{(2n+1)}{U_+}+\frac{1}{R}\overset{(2n)}{U_+}\right)\Bigg\},\tag{22}$$

$$\overset{(2k)}{\sigma_+}=\mu\Bigg\{2\partial_{\bar z}\overset{(2k)}{U_3}-\frac{2k+3}{R}\overset{(2k)}{U_+}+\frac{h}{R}\left[\frac{2k}{4k-1}\left(2\partial_{\bar z}\overset{(2k-1)}{U_3}-\frac{1}{R}\overset{(2k-1)}{U_+}\right)+\right.$$

$$\left.+\frac{2k+1}{4k+3}\left(2\partial_{\bar z}\overset{(2k+1)}{U_3}-\frac{1}{R}\overset{(2k+1)}{U_+}\right)\right]+\frac{2h}{R^2}\left[\frac{k(2k-1)}{4k-1}\overset{(2k-1)}{U_+}-\right.$$

$$\left.-\frac{(k+1)(2k+1)}{4k+3}\overset{(2k+1)}{U_+}\right]+(4k+1)\sum_{n=k}^{N}\left(\frac{h^2+R^2}{hR^2}\overset{(2n+1)}{U_+}+\frac{2}{R}\overset{(2n)}{U_+}\right)\Bigg\},$$

$$\overset{(2k)3}{\sigma}{}_3=\lambda\Bigg\{\overset{(2k)}{\theta}+\frac{2}{R}\overset{2k}{U_3}+\frac{h}{R}\left[\frac{2k}{4k-1}\left(\overset{(2k-1)}{\theta}+\frac{2}{R}\overset{(2k-1)}{U_3}\right)+\right.$$

$$\left.\left.+\frac{2k+1}{4k+3}\left(\overset{(2k+1)}{\theta}+\frac{2}{R}\overset{(2k+1)}{U_3}\right)\right]\right\}+$$

$$+(\lambda+2\mu)\Bigg\{-2\frac{2k+1}{R}\overset{(2K)}{U_3}+\frac{2h}{R^2}\left[\frac{k(2k-1)}{4k-1}\overset{(2k-1)}{U_3}-\frac{(k+1)(2k+1)}{4k+3}\overset{(2k+1)}{U_3}\right]+$$

$$+(4k+1)\sum_{n=k}^{N}\left(\frac{h^2+R^2}{hR^2}\overset{(2n+1)}{U_3}+\frac{2}{R}\overset{(2n)}{U_3}\right)\Bigg\}.$$

$$\overset{(2k+1)}{\sigma_{11}}-\overset{(2k+1)}{\sigma_{22}}+2i\overset{(2k+1)}{\sigma_{12}}=4\mu\Lambda\partial_{\bar z}\overset{(2k+1)_+}{U},$$

$$\overset{(2k+1)\alpha}{\sigma_\alpha}=2(\lambda+\mu)\left(\overset{(2k+1)}{\theta}+\frac{2}{R}\overset{(2k+1)}{U_3}\right)+$$

$$+2\Lambda\left[\frac{2k+1}{R}\overset{(2k+1)}{U_3}+(4k+3)\sum_{n=k+1}^{N}\left(\frac{1}{h}\overset{(2n)}{U_3}+\frac{1}{R}\overset{(2n+1)}{U_3}\right)\right],$$

$$\underset{(+)}{\overset{(2k+1)}{\sigma}}=\mu\left\{2\partial_{\bar z}\overset{(2k+1)}{U_3}+\frac{2k}{R}\overset{(2k+1)}{U_+}+(4k+3)\sum_{n=k+1}^{N}\left(\frac{1}{h}\overset{(2n)}{U_+}+\frac{1}{R}\overset{(2n+1)}{U_+}\right)\right\},$$

$$\overset{(2k+1)}{\sigma_+}=\mu\Bigg\{2\partial_{\bar z}\overset{(2k+1)}{U_3}+\frac{4k+1}{R}\overset{(2k+1)}{U_+}+\frac{h}{R}\left[\frac{2k+1}{4k+1}\left(2\partial_{\bar z}\overset{(2k)}{U_3}-\frac{1}{R}\overset{(2k)}{U_+}\right)+\right.$$

$$\left.+\frac{2k+2}{4k+5}\left(2\partial_{\bar z}\overset{(2k+2)}{U_3}-\frac{1}{R}\overset{(2k+2)}{U_+}\right)\right]+$$

$$+\frac{2h}{R^2}\left[\frac{k(2k+1)}{4k+1}\overset{(2k)}{U_+}-\frac{(k+1)(2k+3)}{4k+5}\overset{(2k+2)}{U_+}+\right.$$

$$+(4k+3)\sum_{n=k+1}^{N}\left(\frac{h^2+R^2}{hR^2}\overset{(2n)}{U_+}+\frac{2}{R}\overset{(2n+1)}{U_+}\right)\Bigg]\Bigg\},$$

$$\overset{(2k+1)_3}{\sigma_{\ 3}}=\lambda\left\{\overset{(2k+1)}{\theta}+\frac{2}{R}\overset{2k+1}{U_3}+\frac{h}{R}\left[\frac{(2k+1)}{4k+1}\left(\overset{(2k)}{\theta}+\frac{2}{R}\overset{(2k)}{U_3}\right)+\right.\right.$$

$$\left.\left.+\tfrac{2k+2}{4k+5}\left(\overset{(2k+2)}{\theta}+\tfrac{2}{R}\overset{(2k+2)}{U_3}\right)\right]\right\}+(\lambda+2\mu)\left\{2\tfrac{2k+1}{R}\overset{(2K+1)}{U_3}+\right.$$

$$+\frac{2h}{R^2}\left[\frac{k(2k+1)}{4k+1}\overset{(2k)}{U_3}-\frac{(k+1)(2k+3)}{4k+5}\overset{(2k+2)}{U_3}\right]+ \qquad (23)$$

$$\left.+(4k+3)\sum_{n=k+1}^{N}\left(\frac{h^2+R^2}{hR^2}\overset{(2n)}{U_3}+\frac{2}{R}\overset{(2n+2)}{U_3}\right)\right\}.$$

The solution of the system (20)-(23) will be sought in the form

$$\overset{(2k)}{U_+}=R^2\partial_{\bar z}(w_{k+1}+i\Omega_{k+1}),$$

$$\overset{(2k+1)}{U_+}=R^2\partial_{\bar z}(w_{N+2+k}+i\Omega_{N+2+k}), \qquad (24)$$

$$\overset{(2k)}{U_3}=hw_{2N+3+k},\quad \overset{(2k+1)}{U_3}=hw_{3N+4+k},$$

$$(k=0,1,...,N).$$

Now, recalling the formula

$$\frac{1}{\Lambda}\frac{\partial}{\partial z}\Lambda\frac{\partial}{\partial\bar z}\frac{1}{\Lambda}\frac{\partial}{\partial\bar z}(\cdot)=\frac{1}{4R^2}\frac{\partial}{\partial\bar z}(\nabla^2+2)(\cdot),$$

where ∇^2 is the Laplacian operator on the surface of a unit sphere, from (20) and (21) we have

$$\nabla^2 w_{k+1}+2\mu k\frac{2k+1}{\lambda+2\mu}w_{k+1}+2\left(\frac{\lambda+3\mu}{\lambda+2\mu}-2k\frac{\lambda+\mu}{\lambda+2\mu}\right)\frac{1}{\delta}w_{2N+3+k}+$$

$$+(4k+1)\left\{\frac{2\lambda}{\lambda+2\mu}\sum_{n=k}^{N}(w_{3N+4+n}+\frac{1}{\delta}w_{2N+3+n})+\frac{\mu}{\lambda+2\mu}\sum_{n=0}^{N}(\delta w_{N+2+n}+\right.$$

$$+w_{n+1})-\frac{2\mu}{\lambda+2\mu}\sum_{n=0}^{k-1}(w_{3N+4+n}+\frac{1}{\delta}w_{2N+3+n})-\frac{\mu}{\lambda+2\mu}\sum_{n=0}^{k-1}[2(2n+1)\times \qquad (25)$$

$$\times\delta w_{N+2+n}+\ (4n+3)\sum_{m=n+1}^{N}\Big((1+\delta^2)w_{m+1}+2\delta w_{N+2+m}\Big)\Big]\Big\}=0,$$

$$\nabla^2 w_{N+2+k}+2(k+1)(2k+1)\frac{\mu}{\lambda+2\mu}w_{N+2+k}+2\left(\frac{3\lambda+5\mu}{\lambda+2\mu}+\right.$$

$$\left.+2k\frac{\lambda+\mu}{\lambda+2\mu}\right)\frac{1}{\delta}w_{3N+4+k}+(4k+3)\left\{\frac{2\lambda}{\lambda+2\mu}\sum_{n=k+1}^{N}(w_{2N+3+n}+\right.$$

$$+\frac{1}{\delta}w_{3N+4+n})+\frac{\mu}{\lambda+2\mu}\sum_{n=0}^{N}(\delta w_{n+1}+w_{N+2+n})- \tag{26}$$

$$-\frac{2\mu}{\lambda+2\mu}\sum_{n=0}^{k-1}(w_{2N+3+n}+\frac{1}{\delta}w_{3N+4+n})-\frac{\mu}{\lambda+2\mu}\sum_{n=0}^{k}[2(2n+1)\delta w_{n+1}-$$

$$-\ (4n+1)\sum_{m=n}^{N}\Big((1+\delta^2)w_{N+2+m}+2\delta w_{m+1}\Big)\Big]\Big\}=0,$$

$$\nabla^2 w_{2N+3+k}+\frac{2(k+1)(2k-1)}{\delta}\frac{\lambda+2\mu}{\mu}w_{2N+3+k}-\left(\frac{\lambda+2\mu}{\mu}+\right.$$

$$\left.+k\frac{\lambda+\mu}{\mu}\right)\delta\nabla^2 w_{k+1}+(4k+1)\left\{\sum_{n=k}^{N}\frac{\delta}{2}(\delta\nabla^2 w_{N+2+n}+\right.$$

$$+\nabla^2 w_{n+1})-\frac{\lambda}{2\mu}\sum_{n=0}^{k-1}\delta(\delta\nabla^2 w_{N+2+n}+\nabla^2 w_{n+1})- \tag{27}$$

$$-\frac{2\lambda}{\mu}\sum_{n=0}^{N}(\delta w_{3N+4+n}++w_{2N+3+n})-\frac{\lambda+2\mu}{\mu}\sum_{n=0}^{k-1}[2(2n+1)\delta w_{3N+4+n}+$$

$$+\ (4n+3)\sum_{m=n+1}^{N}\Big((1+\delta^2)w_{2N+3+m}+2\delta w_{3N+4+m}\Big)\Big]\Big\}=0,$$

$$\nabla^2 w_{3N+4+k}+2k(2k+3)\frac{\lambda+2\mu}{\mu}w_{3N+4+k}+\left(k\frac{\lambda+\mu}{\mu}-1\right)\times$$

$$\times\delta\nabla^2 w_{N+2+k}+(4k+3)\left\{\sum_{n=k+1}^{N}\frac{\delta}{2}(\delta\nabla^2 w_{n+1}+\nabla^2 w_{N+2+n})-\right.$$

$$-\frac{2\lambda}{\mu}\sum_{n=0}^{N}(\delta w_{2N+3+n}+w_{3N+4+n})-\frac{\lambda}{2\mu}\sum_{n=0}^{k}\delta(\delta\nabla^2 w_{n+1}+ \qquad (28)$$

$$+\nabla^2 w_{N+2+n})-\frac{\lambda+2\mu}{\mu}\sum_{n=0}^{k}[-2(2n+1)\delta w_{2N+3+n}+$$

$$\left.+\ (4n+1)\sum_{m=n}^{N}\Big((1+\delta^2)w_{3N+4+n}+2\delta w_{2N+3+n}\Big)\Big]\right\}=0,$$

$$\nabla^2\Omega_{k+1}+2k(2k+1)\Omega_{k+1}+(4k+1)\left\{\sum_{n=0}^{N}(\delta\Omega_{N+2+n}+\Omega_{n+1})-\right.$$

$$-\sum_{n=0}^{k-1}[2(2n+1)\delta\Omega_{N+2+n}+(4n+3)\times \qquad (29)$$

$$\left.\times\sum_{m=n+1}^{N}\left((1+\delta^2)\Omega_{m+1}+2\delta\Omega_{N+2+m}\right)\Big]\right\}=0,$$

$$\nabla^2\Omega_{N+2+k}+2(k+1)(2k+1)\Omega_{N+2+k}+(4k+3)\left\{\sum_{n=0}^{N}(\delta\Omega_{n+1}+\right.$$

$$+\Omega_{N+2+n})-\sum_{n=0}^{k}[-2(2n+1)\delta\Omega_{n+1}+ \qquad (30)$$

$$\left.+(4n+1)\sum_{m=n}^{N}\Big((1+\delta^2)\Omega_{N+2+m}+2\delta\Omega_{m+1}\Big)\Big]\right\}=0,$$

$$\left(\delta=\frac{R}{h}\right).$$

The system of equations (25)-(30) may be written as

$$\sum_{k=1}^{4N+4}(D_{ik}\nabla^2 w_k-L_{ik}w_k)=0,\ (i=1,2,...,4N+4),$$

$$\nabla^2\Omega_i - \sum_{k=1}^{2N+2} B_{ik}\Omega_k = 0,\ (i = 1, 2, ..., 2N+2).$$

The equations may be written in a matrix form

$$D\nabla^2 w - Lw = 0, \tag{31}$$

$$\nabla^2\Omega - B\Omega = 0, \tag{32}$$

where w, Ω are column vectors, respectively, with the components $w_1, ..., w_{4N+4}$ and $\Omega_1, ..., \Omega_{2N+2}; D, L, B$ are matrices with elements D_{ik}, L_{ik}, B_{ik} when D is a triangular matrix.

The matrix equation (31) may be reduced to the form

$$\nabla^2 w - Aw = 0,\ \ A = D^{-1}L.$$

Let the numbers $\alpha_1, ..., \alpha_{4N+4}$ and $\beta_1, ..., \beta_{2N+2}$ be simple eigenvalues of matrices A and B respectively, and let

$$X^{(1)}, ..., X^{(4N+4)}\ and\ Y^{(1)}, ..., Y^{(2N+2)}$$

be their eigenvectors.

The general solutions of matrix equations (31) and (32) have the form

$$w = \sum_{m=1}^{4N+4} X^{(m)}\Phi_m,\ \Omega = \sum_{m=1}^{2N+2} Y^{(m)}\Psi_m, \tag{33}$$

where Φ_m, Ψ_m are arbitrary solutions of the following scalar equations

$$\nabla^2\Phi_m - \alpha_m\Phi_m = 0,\ (m = 1, ..., 4N+4),$$

$$\nabla^2\Psi_m - \beta_m\Psi_m = 0,\ (m = 1, ..., 2N+2).$$

By substituting (33) into (24) we obtain general representations for the components of the displacement vector

$$\overset{(2k)}{U}_+ = R^2\partial_{\bar z}\left(\sum_{m=1}^{4N+4} X_{k+1}^{(m)}\Phi_m + i\sum_{m=1}^{2N+2} Y_{k+1}^{(m)}\Psi_m\right), \tag{34}$$

$$\overset{(2k+1)}{U}_+ = R^2\partial_{\bar z}\left(\sum_{m=1}^{4N+4} X_{N+2+k}^{(m)}\Phi_m + i\sum_{m=1}^{2N+2} Y_{N+2+k}^{(m)}\Psi_m\right), \tag{35}$$

$$\overset{(2k)}{U}_3 = h\sum_{m=1}^{4N+4} X_{2N+3+k}^{(m)}\Phi_m,$$

$$\overset{(2k+1)}{U_3} = h \sum_{m=1}^{4N+4} X_{3N+4+k}^{(m)} \Phi_m .$$

If the substitute formula (34) into relations (22) and (23), we shall obtain general representations for the components of the stress vector.

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