IDENTIFICATION OF THE NONLINEAR DECREASING SYSTEM, REPRESENTED BY VOLTERRA POLYNOMIAL WITH THE SEPARABLE KERNELS

D.I. Bashaleishvili

Ivane Javakhishvili Tbilisi State University
The Faculty of Applied Mathematics and Computer Sciences
2, University str., Tbilisi, 380043, Georgia

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Abstract

The problem of identification of the nonlinear decreasing system, represented by Volterra polynomial with the separable kernels is set. The theorem of the uniqueness of the solution and the theorem with the help of which the problem of nonlinear identification of decreasing system is reduced to the problem of quasilinear identification with short initial data are proved. The set problem of "the black box" identification is solved on the basis of such a projective method's scheme which naturally leads to solution of the system of nonlinear algebraic equations by means of the computer.

Key words and phrases: Identification of the system, decreasing system, increasing system, functional equation, separable kernel, quasilinear system, nonlinear system, the black box.
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The solution of the identification of the decreasing systems (DS) (of crushing and grinding apparatuses) is of great importance in the automatized design problems of technological circuits of departments of crushing and grinding of minerals and other materials of concentrating mills and other industrial works [1].

1. The nonlinear decreasing system, represented by means of Volterra polynomial ("with regard to $f(x)$") of degree n with the separable kernels has the form [1]

$$\varphi(x)=\sum_{i=1}^{n} p_i C_i^{-1}\varphi_1^i(x), \quad x\in[0,a], \quad \varphi(a)=0, \tag{1}$$

where

$$\varphi_1(x)=\int_x^a W_1(x,y)f(y)dy, \quad \varphi_1(a)=0, \tag{2}$$

$$C_i = \int_0^a \varphi_1^i(x)dx, \quad i = \overline{1,n}, \quad C_1 = 1 \tag{3}$$

and

$$p_i \geq 0, \quad i = \overline{1,n}, \quad \sum_{i=1}^{n} p_i = 1. \tag{4}$$

When solving the problem of identification of the nonlinear decreasing system, represented by the substantial functional polynomial, the question of uniqueness of positive solution $\varphi_1(x)$ of equation (1) is solved uniquely by Cartesian theorem (rule) (the number of positive roots is equal or smaller than the number of signs changes in the algebraic polynomial coefficients series by an even number [2]). Indeed, as in expression (1) coefficients are $p_i C_i^{-1} > 0 \ \ \forall_i = \overline{1,n},$ equation

$$\sum_{i=1}^{n} p_i C_i^{-1} \varphi_1^i(x) - \varphi(x) = 0 \quad \forall x \in [0,a] \tag{5}$$

has a unique positive solution $\varphi_1(x)$.

In the identification problem of the studied nonlinear decreasing system the distribution densities of mass by sizes $f(x)$ and $\varphi(x)$, are considered as given, and parameters $p_i, i = \overline{1,n}, C_i, i = \overline{2,n}$ and distribution density $W_1(x,y)$, satysfying conditions $W_1(x,y) \geq 0, \ \int\limits_0^y W_1(x,y)dx = 1$ $\forall y \in [0,a]$ are unknown. The problems of this type are called identification problems of "a black box".

Before solving the set problem of identification of the decreasing system represented by means of Volterra polynomial with separable kernels, it's necessary to discuss the question of uniqueness of solution of identification problem of the quasilinear decreasing system. The above problem is set as the problem of solution of functional equation (1) with $n = 1$ $(p_1 = 1,\ C_1 = 1)$ in regard to function $W_1(x,y)$ with given distribution densities $f(x)$ and $\varphi(x)$ [3,4]. This functional equation solution is sought in the nonlinear set

$$M_\omega = \{W(x,y) \in L_1([0,y] \times [0,a]) : W(x,y) \geq 0,$$
$$\int_0^y W(x,y)dx = 1 \ \forall y \in [0,a]\}.$$

Now let us prove that the solution is unique in class $M_\omega \subset L_1([0,y] \times [0,a])$ if it exists in it. Let us first introduce the function

$$Z(x) = \int_x^a W(x,y)dy, \ x \in [0,a], \ W(x,y) \in M_\omega$$

and set (it will be estimated more precisely below)

$$\widehat{M}_Z = \{Z(x) \in L_1[0,a] : Z(x) \geq 0,\ \int_0^a Z(x)dx = a\}.$$

At first sight $Z(x) = 1$ follows from this expression almost everywhere on $[0,a]$, but in fact this is not the case. Indeed, we have

Theorem 1 *Function* $Z(x) \neq 1$ *almost everywhere on the segment* $[0,a]$.

Proof. Let in expression (1) with $n = 1$ $f(x) = a^{-1}\forall x[0,a]$. Then we have $Z(x) = a\varphi(x)$. Let us set the contrary, that is $Z(x) = 1$ almost everywhere on the segment $[0,a]$. Then $\varphi(x) = a^{-1}\forall x \in [0,a]$ follows from equation $Z(x) = a\varphi(x)$, and this means equivalence (equality) of quasilinear operator (1) when $n = 1$ with the identical operator (it's linear), which contradicts the definition of the quasilinear operator (of the quasilinear decreasing system). The contradiction proves the theorem.

□

The theorem yields the evident

Corollary 1 *Function* $Z(x) \neq const$ *almost everywhere on the segment* $[0,a]$.

Such a set $\widehat{M}_Z$ is defined more precisely by the set

$$M_Z = \{Z(x) \in L_1[0,a] : Z(x) \geq 0,\ Z(x) \not\equiv 1,\ \int_0^a Z(x)dx = a\}.$$

Theorem 2 *Sets* M_ω *and* M_Z *are equivalent* ($M_\omega \sim M_Z$), *that is one-to-one correspondence exists between their elements.*

Proof. The set of nonnegative solutions of the functional equation $\int\limits_0^y W(x,y)dx = 1$ coincides with the set of nonnegative solutions of the functional equation $\int\limits_0^a dy \int\limits_0^y W(x,y)dx = a$ in view of kernel definition $W(x,y) \in M_\omega$.

As kernel $W(x,y) \in M_\omega \subset L_1[0,y] \times [0,a]$, by virtue of Fubini theorem, we have

$$\int_0^a dx \int_x^a W(x,y)dy = \int_0^a dy \int_0^y W(x,y)dx = a.$$

It follows from this that the set of nonnegative solutions of the equation $\int\limits_0^a dy \int\limits_0^y W(x,y)dx = a$ (where the internal integral is equal to unity almost everywhere) coincides with the set of nonnegative solutions of the equation $\int\limits_0^a dx \int\limits_x^a W(x,y)dy = a$ (where by virtue of theorem 1 the internal integral isn't equal to unity almost everywhere). Transitivity is evident, that is, the sets of nonnegative solutions of equations $\int\limits_0^y W(x,y)dx = 1$ and $\int\limits_0^a dx \int\limits_x^a W(x,y)dy = a$ coincide. It is exactly this set which is denoted through M_ω. Now let us denote the last equation as the equation with regard to function $Z(x)$, that is $\int\limits_0^a Z(x)dx = a$. The class of equivalent functions with function $Z(x) \equiv 1$, to which kernel $W(x,y) \in M_\omega$ doesn't correspond, is removed from the set of nonnegative solutions by virtue of theorem 1. The remaining part of the nonnegative solutions makes up the set

$$M_Z = \{Z(x) = \int_x^a W(x,y)dy \in L_1[0,a] : Z(x) \geq 0,\ Z(x) \not\equiv 1,$$
$$\int_0^a Z(x)dx = a\}\,.$$

Such a set M_ω of the nonnegative solutions of the functional equation $\int\limits_0^a dx \int\limits_x^a W(x,y)dy = a$ and set M_Z of the nonnegative solutions of functional equation $\int\limits_0^a Z(x)dx = a$ have identical power as obvious implications $W(x,y) \in M_\omega \Rightarrow \int\limits_x^a W(x,y)dy \in M_Z$ and $\int\limits_x^a W(x,y)dy \in M_Z \Rightarrow$ $\Rightarrow W(x,y) \in M_\omega$ take place. Hence M_ω and M_Z are equivalent sets. The theorem is proved.

□

The next theorems follow from theorem 2 as a consequence

Theorem 3 $W_1(x,y) \overset{a.e.}{=} W_2(x,y) \Rightarrow Z_1(x) \overset{a.e.}{=} Z_2(x)$, *where* $W_1, W_2 \in M_\omega$ *and* $Z_1, Z_2 \in M_Z$.

Theorem 4 *Let on the triangle* $[0,y] \times [0,a]$ *(or on its certain subset of positive measure) almost everywhere* $W_1(x,y) \neq W_2(x,y)$, *then on the segment* $[0,a]$ *(or on its definite subset of positive measure) almost everywhere* $Z_1(x) \neq Z_2(x)$, *where* $W_1, W_2 \in M_\omega$ *and* $Z_1, Z_2 \in M_Z$.

Theorem 5 *Let on the segment* $[0,a]$ *(or on its certain subset of positive measure) almost everywhere* $Z_1(x) \neq Z_2(x)$ *then on the triangle* $[0,y] \times [0,a]$ *(or on its definite subset of positive measure) almost everywhere* $W_1(x,y \neq W_2(x,y)$, *where* $Z_1, Z_2 \in M_Z$ *and* $W_1, W_2 \in M_\omega$.

Theorem 6 $Z_1(x) \overset{a.e.}{=} Z_2(x) \Rightarrow W_1(x,y) \overset{a.e.}{=} W_2(x,y)$, *where* $Z_1, Z_2 \in M_Z$ *and* $W_1, W_2 \in M_\omega$.

Now let us prove the basic theorem of theory of identification of the quasilinear decreasing system.

Theorem 7 *If the functional equation* $\varphi(x) = \int\limits_x^a W(x,y)f(y)dy$ *has a solution* $W_1(x,y)$ *in class* $M_\omega = \{W(x,y) \in L_1([0,y] \times [o,a]) : W(x,y) \geq 0, \ \int\limits_0^y W(x,y)dx = 1 \ \forall y \in [0,a]\}$, *then it is unique with accuracy to the function equivalence.*

Proof. Let us set the contrary, that is, the functional equation has two solutions $W_1(x,y)$ and $W_2(x,y)$, where $W_1(x,y) \neq W_2(x,y)$ almost everywhere on the triangle $[0,y] \times [0,a]$ (or on its certain subset of positive measure). Hence, by virtue of theorem 4, $Z_1(x) \neq Z_2(x)$ almost everywhere on the segment $[0,a]$ (or on its definite subset of positive measure).

As by assumption kernels $W_1(x,y)$ and $W_2(x,y)$ are the solutions of the unknown functional equation, then

$$\int_x^a [W_1(x,y) - W_2(x,y)]f(y)dy = 0, \ x \in [0,a] ,$$

where $f(x)$ is an arbitrary distribution density (evidently, different from the identical zero). Let $f(x) = a^{-1} \forall x \in [o,a]$. Then we have

$$\int_x^a [W_1(x,y) - W_2(x,y)]dy = 0, \ x \in [0,a] ,$$

which contradicts inequality $Z_1(x) \neq Z_2(x)$ and hence, by virtue of theorem 5, assumption $W_1(x,y) \neq W_2(x,y)$. In such a way it's proved that the functional equation

$$h(x) = \int_x^a W(x,y)a^{-1}dy , \ x \in [0,a] ,$$

where $h(x)$ is the distribution density on the quasilinear decreasing system's output when there is an even distribution density $f(x) = a^{-1} \forall x \in [0,a]$ on the input, has a unique solution $W_1(x,y)$. Then for an arbitrary distribution density $f(x)$, different from an even distribution density, there exists a corresponding distribution density $\varphi(x)$, determined by the expression

$$\varphi(x) = \int_x^a W_1(x,y)f(y)dy, \ x \in [0,a] .$$

Let us now consider the quasilinear transformation

$$\alpha_1\varphi(x) + \alpha_2 h(x) = \int_x^a W_1(x,y)[\alpha_1 f(y) + \alpha_2 a^{-1}]dy,$$
$$\alpha_1 > 0,\ \alpha_2 > 0,\ \alpha_1 + \alpha_2 = 1\,.$$

It is evident that, at least, one solution $W_1(x,y)$ of this functional equation exists. Now let us set that besides solution $W_1(x,y)$, there exists another solution $W_2(x,y)$ as well, thereat $W_1(x,y) \neq W_2(x,y)$ almost everywhere on the given triangle or on its certain subset of positive measure. Then from this functional equation for any fixed meaning $\alpha_1 \in (0,1)$ uniquely and simultaneously two equations follow

$$\varphi(x) = \int_x^a W_1(x,y)f(y)dy,\ x \in [0,a]\,,$$

$$h(x) = \int_x^a W_1(x,y)a^{-1}dy,\ x \in [0,a]\,,$$

and each of them, by virtue of assumption, must have two solutions each $W_1(x,y)$ and $W_2(x,y)$. However, according to the above proved the last equation has only one solution $W_1(x,y) = W_2(x,y)$, which contradicts assumption $W_1(x,y) \neq W_2(x,y)$. The contradiction proves the theorem.

□

The set problem of quasilinear decreasing system of identification is equivalent to the problem of the solution of functional equations system

$$\varphi(x) = \int_x^a f(y)W(x,y)dy,$$

$$1 = \int_0^y W(x,y)dx$$

with regard to the function $W(x,y)$. It is solved by the projective method, which reduces the problem of solution of functional equations system to the problem of solution of linear algebraic equations system [3,4]. We want to extend the same method only in a changed form to the problem of identification of essentially nonlinear decreasing systems represented through (but not in a form) Volterra polynomial with the separable kernels (this class of nonlinear decreasing system is narrowing of Volterra polynomial determined on any linear limited subset R of Banach space $C[0,a]$ or $L_p[0,a]$, $p \in [1,\infty]$, on the nonlinear subset $M_f = \{f(x) \in F : f(x) \geq 0,\ \int_0^a f(x)dx = 1\} \subset R$, where F is a Banach space.

Proposition takes place

Proposition 1 *If the function* $W_1(x,y)$ *is the solution of functional equations system (a problem of the nonlinear decreasing system identification)*

$$\begin{aligned} &\varphi(x) = \sum_{i=1}^{n} p_i C_i^{-1} \Big(\int\limits_x^a W_1(x,y) f(y) dy \Big)^i, \\ &\int\limits_0^y W_1(x,y) dx = 1, \end{aligned} \tag{6}$$

then the same function $W_1(x,y)$ *is a unique solution of functional equations system (a problem of the quasilinear decreasing system identification)*

$$\begin{aligned} &\varphi_1(x) = \int_x^a W_1(x,y) f(y) dy, \\ &\int_0^y W_1(x,y) dx = 1. \end{aligned} \tag{7}$$

Proof. Let functional equations system (6) have a solution $W_1(x,y)$. Then we have the identity

$$\varphi(x) \equiv \sum_{i=1}^{n} p_i C_i^{-1} \Big(\int_x^a W_1(x,y) f(y) dy \Big)^i. \tag{8}$$

As the function $W_1(x,y)$ is the solution of the second functional equation of system (6), the function $\varphi_1(x) \equiv \int\limits_x^a W_1(x,y) f(y) dy$ satisfies condition $\int\limits_0^a \varphi_1(x) dx = 1$. Then identity (8) has the form

$$\varphi(x) \equiv \sum_{i=1}^{n} p_i C_i^{-1} \varphi_1^i(x),$$

which means uniqueness of the positive solution $\varphi_1(x)$ of equation (5). Hence, the function $\varphi_1(x)$ is a distribution density. Since it is the distribution density $\varphi_1(x)$ which appears in problem (7), the function $W_1(x,y)$ is the unique solution of system (7), by virtue of theorem 7. The theorem is proved.

□

The contrary also takes place

Proposition 2 *If the function* $W_1(x,y)$ *is the solution of functional equations system (7)(it's unique), then the function* $W_1(x,y)$ *is the unique solution of functional equations system (6).*

Proof. As distribution density $\varphi_1(x)$ is the unique positive solution of equation (5), and corresponding to it identification quasilinear problem has a unique solution $W_1(x,y),$, distribution density $\varphi_1(x) = \int\limits_x^a W_1(x,y)f(y)dy$ converts equation (5) into identity (8). It means that the function $W_1(x,y)$ is the solution of functional equations system (6). In such a way, functional equations systems (6) and (7) have one and the same solution $W_1(x,y)$, while system (7) has no other solutions, by virtue of theorem 7.

Let system (6) have another solution $\widetilde{W}_1(x,y)$ too. Then the first equation of system (6) converts into identity

$$\varphi(x) \equiv \sum_{i=1}^{n} p_i C_i^{-1} \Big(\int_x^a \widetilde{W}_1(x,y)f(y)dy \Big)^i. \qquad (9)$$

From comparison of expressions (8) and (9), by virtue of distribution density $\varphi_1(x)$ uniqueness, we have

$$\int_x^a W_1(x,y)f(y)dy = \int_x^a \widetilde{W}_1(x,y)f(y)dy.$$

It means that the function $\widetilde{W}_1(x,y)$ is the second solution of system (7), which is impossible. The obtained contradiction proves uniqueness of solution $W_1(x,y)$ of system (6).

□

Proved propositions 1 and 2 admit to replace nonlinear identification problem solution (6) by quasilinear identification simpler problem (7). However, in problem (7) distribution density $\varphi_1(x)$ isn't given (distribution density $\varphi(x)$ is given). Therefore representation problem $\varphi_1(x)$ arises through the given distribution density $\varphi(x)$. The possibility of this representation is given by the solution of equation (5) with regard to $\varphi_1(x)$. Though equation (5) has always a unique positive solution – distribution density $\varphi_1(x)$ for any finite values of n, the solution $\varphi_1(x)$ is expressed in radicals only when $n = 2, 3, 4$.

Let us confine ourselves to the case $n = 2$. Equation (5) takes the form of the quadratic equation

$$p_2 C_2^{-1} \varphi_1^2(x) + p_1 \varphi_1(x) - \varphi(x) = 0 \qquad (10)$$

with regard to $\varphi_1(x)$. As for $p_1 \in [0,1)$, $C_2^{-1} > 0$ and $\varphi(x) \geq 0 \ \ \forall x \in [0,a]$ discriminant $D(x) = -4(1-p_1)C_2^{-1}\varphi(x) - p_1^2 < 0$, equation (10) has a unique positive solution

$$\varphi_1(x) = \frac{+\sqrt{p_1^2 + 4(1-p_1)C_2^{-1}\varphi(x)} - p_1}{2(1-p_1)C_2^{-1}} \qquad (11)$$

is a distribution density. Hence, by virtue of propositions 1 and 2, nonlinear identification problem (6) with $n = 2$ is replaced by quasilinear identification problem

$$\frac{+\sqrt{p_1^2 + 4(1-p_1)C_2^{-1}\varphi(x)} - p_1}{2(1-p_1)C_2^{-1}} = \int_x^a W_1(x,y)f(y)dy, \qquad (12)$$
$$\int_x^a W_1(x,y)dx = 1, \quad \forall y \in [0,a].$$

2. The theorems of the previous section allow to solve the problem of identification of the decreasing system, represented through Volterra polynomial with the separable kernels to the fourth degree inclusive since distribution densities $\varphi_1(x)$ are represented by radicals from unknown parameters and the known distribution density $\varphi(x)$. For the value of $n = 1$, that's for $\varphi_1(x) = \varphi(x)$, the solution method was determined by theorem given in the earlier article of the author [4]. The solution of identification problem is reduced to the finite-dimensional problem of solution of the system of linear algebraic equations

$$\sum_{j=1}^{k} C_{ij} g_j = b_i, \quad i = \overline{1,k}, \qquad (13)$$

where

$$C_{ij} = \int_0^a \psi_i(x)dx \int_x^a \omega_j(x,y)f(y)dy = \int_0^a f(y)dy \int_0^y \psi_i(x)\omega_j(x,y)dx,$$
$$i = \overline{1,k}, \quad j = \overline{1,k},$$
$$b_i = \int_0^a \psi_i(x)\varphi_1(x)dx, \quad i = \overline{1,k},$$

and $\psi_i(x)$, $i = \overline{1,k}$ *and* $\omega_j(x,y)$, $j = \overline{1,k}$ are given systems of linearly independent distribution densities [3,4]. The same scheme of the projective method may be used in cases $n = 2, 3, 4$ too. The matter is, that the elements of matrix (C_{ij}) don't depend on unknown parameters p_i, $i = \overline{1, n-1}$ and C_i, $i = \overline{2,n}$. Only the components of vector $b = (b_1, b_2, \ldots, b_k)$, depend on these parameters, that is

$$b_i = \chi_i(p_1, p_2, \ldots, p_{n-1}, C_2, C_3, \ldots, C_n), \quad i = \overline{1,k}.$$

Consequently, it's necessary to increase the number of equations k in expression (13) by number of unknown parameters $2n - 2$ to find g_j, $j =$

$=\overline{1,k}$ and $p_1,p_2,\ldots,p_{n-1}$, $C_2,C_3,\ldots,C_n$. Then instead of expression (13) the nonlinear equations system will be

$$\begin{aligned}&\sum_{j=1}^{k} C_{ij}g_j-\chi_i(p_1,p_2,\ldots,p_{n-1},C_2,C_3,\ldots,C_n)=0,\\&i=\overline{1,k+2n-2},\end{aligned}\tag{14}$$

where

$$\begin{aligned}&\chi_i(p_1,p_2,\ldots,p_{n-1},C_2,C_3,\ldots,C_n)=\\&=\int_0^a\psi_i(x)\widetilde{\varphi}_1(x,p_1,p_2,\ldots,p_{n-1}C_2,C_3,\ldots,C_n)dx,\end{aligned}\tag{15}$$

and $\varphi_1(x)=\widetilde{\varphi}_1(x,p,C)$ (it is determined by expression (11) when $n=2$).

When $n=2$, expression (14), is given in the form

$$\begin{aligned}&C_{11}g_1+C_{12}g_2+\ldots+C_{1k}g_k-\chi_1(p_1,C_2^{-1})=0,\\&C_{21}g_1+C_{22}g_2+\ldots+C_{2k}g_k-\chi_2(p_1,C_2^{-1})=0,\\&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\\&C_{k1}g_1+C_{k2}g_2+\ldots+C_{kk}g_k-\chi_k(p_1,C_2^{-1})=0,\\&C_{k+1\ 1}g_1+C_{k+1\ 2}g_2+\ldots+C_{k+1\ k}g_k-\chi_{k+1}(p_1,C_2^{-1})=0,\\&C_{k+2\ 1}g_1+C_{k+2\ 2}g_2+\ldots+C_{k+2\ k}g_k-\chi_{k+2}(p_1,C_2^{-1})=0.\end{aligned}$$

By comparison with problem (13) $k(2n-2)$ of elements C_{ij}, $j=\overline{1,k}$; $i=\overline{k+1,k+2n-2}$ and $2n-2$ of components $b_{k+1},b_{k+2},\ldots,b_{k+2n-2}$ of vector b are computed in addition in problem (14), where the calculations of $b_i=\chi_i$ become complicated.

Nonlinear equations system (14) is solved by global convergent modifications of Newton method [5,6,7], for example, by Levenberg-Markvard method as it is fit not only for convex functions, and convergence is guaranted independently from the initial approximation. Programme "MathCad" is used for calculation of matrix elements (C_{ij}) as well as for function χ_i. Finally the approximation solution of nonlinear descreasing identification problem takes the form

$$W_1^{(k)}(x,y)=\sum_{j=1}^{k}\widehat{g}_j\omega_j(x,y),$$

and the decreasing system mathematical model is

$$\widetilde{\varphi}(x)=\sum_{i=1}^{n}\widehat{p}_i\widehat{C}_i^{-1}\Big(\int_x^a W_1^{(k)}(x,y)f(y)dy\Big)^i,\quad n=2,3,4,$$

where $\widehat{g}_j$, $j=\overline{1,k}$, $\widehat{p}_1, \widehat{p}_2, \ldots, \widehat{p}_{n-1}, \widehat{C}_2^{-1}, \widehat{C}_3^{-1}, \ldots, \widehat{C}_n^{-1}$ are the components of the approximate solution of equations system (14), and $C_1^{-1}=1$ and $\widehat{p}_n = 1 - \sum\limits_{i=1}^{n-1} \widehat{p}_i$.

The accuracy of nonlinear identification problem is estimated by relative error ε of computing the element $\widetilde{\varphi}(x) \in M_\varphi$

$$\varepsilon = \frac{||\varphi(x) - \widetilde{\varphi}(x)||}{||\varphi(x)||}.$$

Analogously, the problem of identification of nonlinear increasing system (IS), represented by Volterra polynomial with the separable kernels [1] may be solved .

At last it should be noted that identification theory of the studied static systems which has been cited above in [4] may be used not only for the pointed technological processes research, but for solution of some problems in many branches of natural sciences as well (physics, chemistry, astronomy, biology, medicine and so on; for example, the problem of citologic diagnostics of oncologic and other diseases if by $f(x)$ and $\varphi(x)$ we mean distribution densities of nuclei number of sells in their size in norm and pathology accordingly and by operators of DS and IS – the form of a disease, and so on).

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